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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

THE PASSAGE OF TURBID WATER THROUGH LAKE MEAD

By Nathan C. Grover,[1] M. Am. Soc. C. E., and
Charles S. Howard,[2] Esq.

Synopsis

Turbid water carrying a considerable load of fine silt was discharged from Lake Mead, above Boulder Dam, in Arizona and Nevada, at three different periods during 1935 when the reservoir was 70 to 90 miles long and contained from 4 000 000 to 5 000 000 acre-ft of water. Apparently, it flowed through the reservoir essentially unmixed. Chemical analyses of the water entering into, and discharged from, the reservoir corroborate the conclusions drawn from the observations of silt as to the occasional discharge of essentially unmixed water. The phenomenon is ascribed to the greater specific gravity of the incoming water relative to the generally clear water at the surface of the lake, due probably, in part, to its silt load. A practical significance is suggested with respect to the possibility of increasing the passage of fine silt through a reservoir, thereby prolonging its effective life. References are made to other known occurrences of similar phenomena.

Introduction

The apparent resistance of water to mixing, under certain conditions, is a matter of common knowledge based on many observations. A river having water of one color receives a tributary having water of another color, and the two waters flow for miles side by side in the same channel without very much mixing. The Gulf Stream flows for hundreds of miles through the Atlantic Ocean and retains such definiteness of boundaries (identified by color, temperature, and organic matter), that a ship's bow may be situated within, and its stern outside, the stream. The late John R. Free-

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Note.—By decision of February 1, 1936, by the Board on Geographical Names, the reservoir formed by Boulder Dam on Colorado River in Arizona and Nevada, was named Lake Mead, in honor of the late Elwood Mead, M. Am. Soc. C. E. This paper is published by permission of the Director, U. S. Geological Survey. Discussion on this paper will be closed in **August, 1937,** *Proceedings*.

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[2] Chemist, U. S. Geological Survey, Washington, D. C.

man,[3] Past-President and Hon. M. Am. Soc. C. E., calls attention to rivers that discharge turbid water into the clear water of lakes. The phenomenon may be seen as the current cascades to considerable depths practically unmixed with the clear lake waters through which it is falling. The flowing of one water through, or by the side of, another water without rapid mixing might be illustrated by many examples.

There is one type of the failure of water to mix which has not been so commonly observed and has not been adequately recorded in engineering literature, but which may be of great interest and perhaps practical value to engineers. This type relates to the flow of turbid water through a reservoir of great length and its discharge at the dam with its conspicuous characteristics unchanged. Obviously, as the discharge of turbid water from a reservoir does not occur at all times when such water is flowing into it, there must be certain conditions of the inflowing water related to its specific gravity, viscosity, temperature, size, and degree of dispersion of the silt particles (or other physical qualities and combinations of them), that are prerequisite to such discharge. Although such discharges have been mentioned occasionally in engineering literature, the conditions under which they occur have not, as far as the writers are informed, been thoroughly described or studied. Mr. H. F. Robinson[4] quotes a report made in 1914 by Rollin Ritter, Assoc. M. Am. Soc. C. E., that "the silt-laden waters, being heavier than the clear, oftimes appeared at the tunnel outlet when the reservoir surface was clear and a small or moderate flood was entering the reservoir." Such a flow of turbid water through a long reservoir has been observed at Lake Murray, about 30 miles long, on the Saluda River, in South Carolina. L. M. Lawson,[5] M. Am. Soc. C. E., has mentioned the discharge of silt through the gates of Elephant Butte Dam on the Rio Grande, in New Mexico. Mr. L. R. Fiock[6] states that flows of turbid water from the Elephant Butte Reservoir, the head of which has been 30 to 40 miles above the dam, have occurred in 7 yr out of 20 yr of its operation. The movement of turbid water through the lake formed by Boulder Dam has also been mentioned.[7]

Measurements by Mr. H. M. Eakin,[8] of the sediments deposited in many reservoirs, appear to indicate similar flows of unmixed waters through reservoirs of various lengths and characteristics. He reports that his measurements disclose two types of deposition of sediment in a reservoir—in delta areas adjacent to points of stream inflow and in the deeper parts of the reservoir. The deposition of sediment in the deep parts near the dam are explained by him on the hypothesis that turbid water passed through the reservoir from the point of entrance.

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[3] "Hydraulic Laboratory Practice", John R. Freeman, Editor, p. 322, A. S. M. E., 1929.

[4] "The Silt Problem of the Zuni Reservoir", Discussion by H. F. Robinson, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 873.

[5] "Movement of Silt, Elephant Butte Reservoir", by L. M. Lawson, *Reclamation Record*, Vol. 10 (1919), p. 411.

[6] *Transactions*, Am. Geophysical Union, 1934, Pt. 2, p. 472.

[7] *Reclamation Era*, October, 1935.

[8] "Silting of Reservoirs", by Henry M. Eakin, *Technical Bulletin 524*, U. S. Dept. of Agriculture, Soil Conservation Service, p. 7.

COLORADO RIVER AND LAKE MEAD

The Colorado River normally carries large quantities of silt. Under natural conditions this silt moves down the river channel, in part continuously and in part by stages of alternate deposition and erosion. Except as modified by gradual degradation or aggradation of the channel, as much silt would normally pass one point on the river over a considerable period of time as would pass another point, due allowance being made for accretions from tributaries; but within short periods there would not be such equality of silt movement.

Lake Mead was created by the construction of Boulder Dam, about 600 ft high, above the stream bed (731 ft from the lowest point in the foundation to the highest point of the dam), in the Black Canyon of the Colorado River. The lake thus formed has an initial capacity, at spillway level, of 30 500 000 acre-ft, which is about the total run-off of the river in two average years. When the reservoir is filled, the lake will have a length of about 115 miles and a surface area of about 145 000 acres. It is irregular not only in outline but in cross-section. Within it are several box, or near-box, canyons through which the Colorado River flows, ranging from 2 to 10 miles in length, separated by more open valleys into which the lake widens. The lake, therefore, may be considered as composed of several great pools, notably at or near Las Vegas Wash, the Virgin River, Hualpai Wash, and Grand Wash, separated by the relatively narrow necks formed by the canyons. The making of this lake has changed the natural processes of the river within the 115 miles of the lake and, of course, throughout the course of the river below the lake. The annual floods in the river below the lake, with their eroding and transporting action, have been eliminated. In this part of the river the range in stage and discharge is now relatively small. The conditions in the basin above the lake are unchanged, of course, and most of the silt that enters the lake is now (1937) deposited and remains there.

After the filling of the lake had progressed for a short time, the effluent became essentially clear, and, in general, it has continued so since. However, turbid water was discharged from Lake Mead three times during the eleven months from February 1, 1935, when storage began, to December 31, 1935, although the quantity flowing through the openings in the dam throughout the period has been controlled at less than 10 000 cu ft per sec. During these months the water was flowing through the tunnels at the base of the dam, and, when the turbid discharges occurred, the lake was from 70 to 90 miles long and contained from 4 000 000 to 5 000 000 acre-ft of water; there was no significant change in the quantity of outflow. The turbid discharges represent the conveyance through, and the discharge from, the lake of as much as 6 000 000 tons of silt. At these times there has been not only the ocular evidence of unmixed or partly mixed water flowing through the lake, as disclosed by its turbidity, but this evidence has been substantiated by measurements of the size of the particles of suspended matter and by the determination of the dissolved matter, as indicated by the sulfates, contained in the water entering into, and discharged from, the lake.

MEASUREMENTS AND OBSERVATIONS

The loads of suspended matter carried by the Colorado River are computed for the Grand Canyon Gaging Station, about 265 miles above Boulder Dam, and for the Willow Beach and Topock Gaging Stations, about 10 and 115 miles, respectively, below the dam. The positions of these stations with respect to each other and to the lake are shown by Fig. 1. The percentages by weight of suspended matter are given in Table 1 for the Grand Canyon, Willow Beach, and Topock Gaging Stations during the period, March 1 to October 30, 1935. The loads, in tons per day, are shown in Fig. 2 (*c*) for the Grand Canyon and Willow Beach stations.

Comparisons of the loads carried past the Grand Canyon and Willow Beach Stations, give an approximate measure of the quantity of material deposited in the lake. Comparisons of the loads carried past the Willow Beach and Topock Stations give a measure of the scouring of the river channel that occurs between those two stations. The results for the Topock

TABLE 1.—SUSPENDED MATTER (PERCENTAGE BY WEIGHT), IN COLORADO RIVER, GAGING STATIONS DURING THE PERIOD

Day of month	MARCH			APRIL			MAY			JUNE		
	G.C. (1)	W.B. (2)	Top. (3)	G.C. (4)	W.B. (5)	Top. (6)	G.C. (7)	W.B. (8)	Top. (9)	G.C. (10)	W.B. (11)	Top. (12)
1	0.46	0.09	0.37	0.93	0.07	0.44	0.85	0.04	0.26	1.70	0.06	0.32
2	0.44	0.10	0.39	1.06	0.06	0.35	0.90	0.04	0.25	1.90	0.06	0.30
3	0.48	0.10	0.40	1.26	0.06	0.31	1.04	0.03	0.28	1.58	0.04	0.28
4	0.35	0.07	0.39	1.24	0.06	0.32	1.22	0.03	0.26	1.45	0.07	0.28
5	0.30	0.07	0.35	1.31	0.07	0.27	1.28	0.03	0.22	1.31	0.06	0.29
6	0.28	0.05	0.40	1.38	0.07	0.30	1.04	0.03	0.24	1.12	0.05	0.30
7	0.32	0.05	0.32	1.40	0.14	0.25	0.99	0.05	0.22	1.08	0.05	0.26
8	0.44	0.06	0.34	1.48	0.32	0.25	1.06	0.04	0.21	1.10	0.05	0.28
9	0.46	0.08	0.36	1.42	0.38	0.40	0.88	0.04	0.21	1.28	0.04	0.29
10	0.54	0.06	0.55	1.60	0.53	0.46	0.73	0.04	0.20	1.28	0.06	0.34
11	0.44	0.06	0.44	3.16	0.95	0.46	0.68	0.05	0.20	1.25	0.05	0.24
12	0.41	0.06	0.40	2.16	0.70	0.66	0.68	0.05	0.19	1.26	0.06	0.32
13	0.35	0.06	0.42	1.58	0.72	0.82	0.77	0.07	0.24	1.14	0.05	0.28
14	0.38	0.06	0.30	1.31	0.95	0.77	0.80	0.10	0.27	1.22	0.04	0.26
15	0.36	0.06	0.38	1.12	0.81	0.99	0.98	0.06	0.22	1.06	0.04	0.32
16	0.43	0.05	0.44	0.89	1.15	1.00	1.13	0.06	0.19	1.09	0.04	0.31
17	0.44	0.05	0.38	0.81	0.96	1.00	1.45	0.06	0.22	1.03	0.04	0.28
18	1.72	0.05	0.32	0.84	0.62	1.24	1.52	0.07	0.27	1.01	0.06	0.27
19	1.55	0.06	0.43	1.11	0.45	0.91	1.36	0.05	0.25	0.82	0.04	0.28
20	1.16	0.03	0.30	1.46	0.24	0.70	1.32	0.05	0.24	0.79	0.04	0.26
21	0.94	0.04	0.31	1.38	0.31	0.60	1.26	0.04	0.27	0.72	0.05	0.28
22	0.89	0.05	0.33	1.38	0.04	0.54	1.22	0.06	0.20	0.55	0.04	0.32
23	0.79	0.04	0.30	1.23	0.04	0.42	1.38	0.08	0.23	0.54	0.04	0.32
24	0.70	0.06	0.32	1.20	0.05	0.48	1.48	0.07	0.40	0.58	0.04	0.30
25	0.61	0.16	0.27	1.10	0.04	0.30	1.48	0.08	0.47	0.56	0.04	0.24
26	0.60	0.52	0.28	1.06	0.40	0.26	1.08	0.08	0.45	0.44	0.05	0.32
27	0.64	0.41	0.57	1.26	0.08	0.26	1.01	0.09	0.35	0.48	0.05	0.23
28	0.61	0.34	0.70	1.40	0.05	0.57	1.01	0.09	0.38	0.44	0.05	0.26
29	0.56	0.18	0.57	1.08	0.05	0.26	1.26	0.06	0.30	0.37	0.04	0.26
30	0.59	0.30	0.58	0.92	0.06	0.29	1.38	0.07	0.38	0.36	0.07	0.27
31	0.72	0.31	0.54	....	....	....	1.54	0.06	0.30	....	....	....

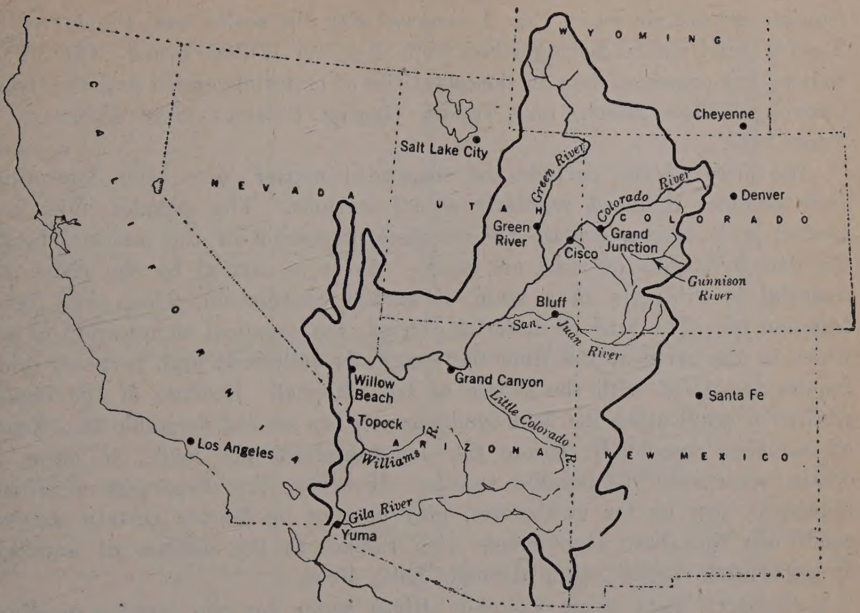

FIG. 1.—MAP OF COLORADO RIVER DRAINAGE BASIN.

AT THE GRAND CANYON (G.C.), WILLOW BEACH (W.B.), AND TOPOCK (TOP.) MARCH 1 TO OCTOBER 31, 1935.

Day of month	July			August			September			October		
	G.C. (13)	W.B. (14)	Top. (15)	G.C. (16)	W.B. (17)	Top. (18)	G.C. (19)	W.B. (20)	Top. (21)	G.C. (22)	W.B. (23)	Top. (24)
1	0.36	0.04	0.26	0.16	0.04	0.18	1.43	0.03	0.14	5.09	0.005	0.26
2	0.35	0.05	0.29	0.25	0.03	0.20	1.83	0.02	0.14	4.65	0.007	0.21
3	0.33	0.06	0.26	0.17	0.05	0.21	2.80	0.86	0.18	3.44	0.02	0.27
4	0.28	0.07	0.36	0.23	0.03	0.22	5.02	0.99	0.24	1.98	0.007	0.24
5	0.28	0.04	0.26	0.36	0.04	0.16	2.54	0.68	0.92	1.28	0.01	0.20
6	0.30	0.05	0.26	1.82	0.04	0.16	1.94	0.17	0.91	0.88	1.57	0.28
7	0.22	0.08	0.26	1.54	0.05	0.28	2.00	0.42	0.78	0.69	1.60	0.58
8	0.23	0.04	0.24	0.72	0.04	0.39	1.64	0.94	0.40	0.54	1.64	1.62
9	0.23	0.05	0.26	0.44	0.05	0.20	1.12	0.70	0.81	0.45	1.99	1.71
10	0.20	0.04	0.26	0.46	0.04	0.16	0.97	0.57	1.00	0.42	2.16	1.79
11	0.17	0.04	0.26	1.72	0.07	0.21	0.72	1.08	0.92	0.31	1.86	2.12
12	0.19	0.03	0.18	1.13	0.05	0.22	0.64	1.37	0.77	0.26	1.19	2.34
13	0.16	0.03	0.18	0.97	0.04	0.17	0.56	0.71	1.03	0.22	0.34	2.00
14	0.16	0.02	0.17	1.11	0.04	0.22	0.65	0.27	1.33	0.19	0.004	1.24
15	0.16	0.03	0.21	1.16	0.01	0.25	0.54	0.01	0.79	0.16	0.002	0.51
16	0.15	0.04	0.23	0.98	0.02	0.17	0.56	0.03	0.28	0.14	0.005	0.30
17	0.13	0.01	0.16	0.76	0.02	0.19	0.40	0.01	0.32	0.13	0.009	0.25
18	0.13	0.01	0.19	1.35	0.03	0.23	0.33	0.02	0.22	0.12	0.003	0.28
19	0.14	0.03	0.16	0.53	0.02	0.24	0.41	0.01	0.20	0.11	0.02	0.26
20	0.30	0.02	0.20	0.40	0.03	0.18	0.31	0.01	0.21	0.11	0.02	0.27
21	0.39	0.04	0.20	0.44	0.02	0.20	0.21	0.01	0.22	0.12	0.02	0.31
22	0.62	0.04	0.21	0.54	0.03	0.25	0.16	0.03	0.24	0.12	0.02	0.30
23	0.60	0.04	0.22	0.48	0.04	0.15	0.15	0.01	0.20	0.11	0.01	0.34
24	0.35	0.04	0.23	0.50	0.02	0.14	0.14	0.01	0.24	0.12	0.01	0.40
25	0.24	0.03	0.26	0.52	0.02	0.16	0.12	0.01	0.23	0.14	0.01	0.26
26	0.27	0.03	0.26	4.81	0.02	0.15	0.15	0.01	0.20	0.15	0.01	0.27
27	0.54	0.03	0.40	2.56	0.02	0.15	0.24	0.01	0.22	0.16	0.01	0.28
28	0.46	0.02	0.23	2.74	0.04	0.15	0.90	0.01	0.26	0.20	0.02	0.26
29	0.23	0.02	0.31	1.86	0.02	0.19	2.14	0.02	0.26	0.49	0.02	0.27
30	0.22	0.03	0.24	1.69	0.02	0.13	4.14	0.01	0.21	0.44	0.02	0.24
31	0.19	0.03	0.22	2.14	0.02	0.14	....	....	....	0.26	0.01	0.33

Station are not shown in Fig. 2, because with the scales used the curves for Topock could not be distinguished from those for Willow Beach. One of the writers[9] has presented data on the quantities of material carried past the Grand Canyon, Willow Beach, and Topock Gaging Stations from February to June, 1935.

The sizes of the particles of suspended matter have been determined from samples collected regularly at all stations. The samples have been treated with sodium oxalate for complete dispersion of the material before the determinations of sizes are made. As it is carried by the river, the material is evidently in a state of partial coagulation, which may be a function of: The kind of material carried; the chemical composition of the water in the river at the time the sample is collected; and, perhaps, other factors associated with the source of the material. Because of the impossibility of duplicating the field conditions, it has seemed desirable to disperse all samples, completely, before the determinations are made, in order to obtain reasonably comparable results. However, the dispersion, desirable though it may be for uniformity, may obscure or destroy certain natural conditions that have significance with respect to the passage of unmixed or imperfectly mixed water through Lake Mead.

Laboratory tests using Colorado River water for the settling medium show that a large percentage of the undispersed material has a much faster settling rate than the same material in a state of complete dispersion. It now appears, therefore, that it would be desirable to know the sizes of the particles as they are actually carried in suspension, inasmuch as it is evidently such sizes that may be most significant with respect to the passage of turbid water through a reservoir. It is probable that determinations should be made of the sizes of the material both as received from the river and after complete dispersion.

The larger sizes of particles have been measured and separated by sieves. However, if a considerable proportion of such matter passes a 200-mesh sieve (diameter of particles 74 microns), further determinations of size are made by observing times of settling in tall cylinders and computing sizes by Stokes' law. The settling method, of course, is not exact, because the separations represent quantities of particles that settle at given rates rather than quantities of particles that have given diameters, and each size group obtained contains particles that are smaller than the size for which the separation was made and that have been carried down with the larger sizes. According to Stokes' law the settling rates are, approximately, 26 ft, 4 ft, and 0.3 ft per hr for 50-micron, 20-micron and 5-micron sizes, respectively.

Fig. 2(*c*) shows the daily load of suspended matter at the Grand Canyon and Willow Beach Gaging Stations during the year ending December 31, 1935. During January, 1935, before the flow of the river was regulated by the dam, the load of suspended matter at the Willow Beach Station was

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[9] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 284.

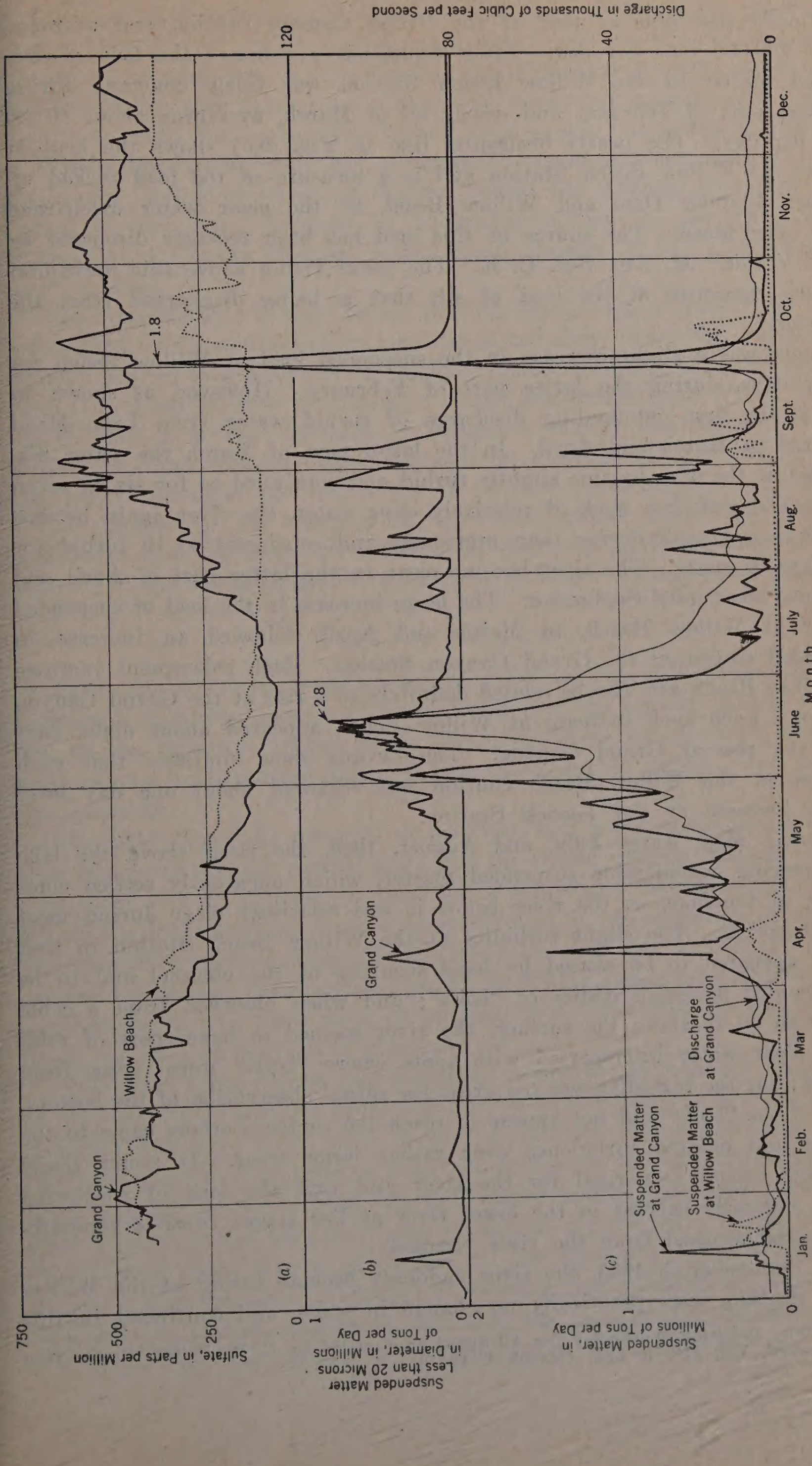

FIG. 2.—SULFATE SUSPENDED MATTER, AND DISCHARGE AT GAGING STATIONS ON COLORADO RIVER NEAR GRAND CANYON AND WILLOW BEACH, ARIZONA.

practically the same as that at the Grand Canyon Station and averaged about 100 000 tons per day. After regulation was begun the load of suspended matter at the Willow Beach Station was fairly constant during the first part of February and nearly all of March, averaging about 10 000 tons per day. The nearly horizontal line in Fig. 2(*c*) shows the load of silt at the Willow Beach Station and is a measure of the load picked up between Boulder Dam and Willow Beach by the clear water discharged from Lake Mead. The source of this load has been recently discussed by C. P. Vetter,[10] M. Am. Soc. C. E. The peaks rising above this horizontal line are measures of the load of silt that is being discharged from the reservoir.

There was a slight increase in the suspended load at Willow Beach for a few days, during the latter part of February. However, as shown in Fig. 2, the first outstanding discharge of turbid water from Lake Mead occurred in March and April. In the latter part of March the water discharged at the dam became slightly turbid and continued so for six or seven days. Then, after a week of relatively clear water, the river again became turbid to a greater degree than previously and continued to be turbid for ten days or more. The river became clear in the latter part of April and remained clear until September. The large increase in the load of suspended matter at Willow Beach, in March and April, followed an increase in suspended matter at the Grand Canyon Station. Each subsequent increase at Willow Beach can also be related definitely to a rise at the Grand Canyon Station. Each such increase at Willow Beach appeared about eight days after the rise at Grand Canyon. The records show similarly that each increase at the Willow Beach Station was followed about one day later by an increase at the Topock Station.

During May, June, July, and August, 1935, the river above the lake was carrying considerable suspended matter, which apparently settled completely in the lake, as the river below it was relatively clear during most of that period. The slight turbidity at the Willow Beach Station in that period appeared to be caused by local scouring of the channel and to be accompanied by small eddies or "boils"; and when observed from a cable car 50 to 60 ft above the surface, the river seemed to have spots of relatively clear water interspersed with spots where "boils" were rising from the bed; but the turbidity was too great for actual observation of the bottom. Some of the "boils" did not appear to reach the surface; others came to the surface and created turbulence over rather large areas. It seems likely that this action is normal for the river and that the load of suspended matter normally carried in the lower river at low stages consists generally of material scoured from the river channel.

On September 3, 1935, the river suddenly became turbid at the Willow Beach Station, with practically no change in stage, and continued in this condition until the morning of September 14, when it was again practically clear. During this 12-day period more than 2 000 000 tons of suspended matter was carried past Willow Beach. On September 5, 1935, the river at the Topock Station showed an increase in suspended matter, and for the next twelve days the load at the Topock Station was comparable to the load at Willow Beach, but there was a marked decrease on September 16. The appearance of turbid water at Willow Beach and Topock apparently was related to the large increase in suspended matter in the river water at the Grand Canyon Station, which began August 26 and continued for fourteen days.

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[10] "Why Desilting Works for the All-American Canal"? by C. P. Vetter, *Engineering News-Record,* Vol. 118, p. 321, (March 4, 1937).

The river at the Willow Beach Station again became turbid on October 6, 1935, and remained in that condition for eight days. On October 14, it had returned to a condition of slight turbidity. During this 8-day period more than 2 500 000 tons of suspended matter was carried past Willow Beach. On October 8, a large increase in suspended matter was observed at the Topock Station, followed by a decrease on October 14 and a return on October 15 to about the conditions of the first seven days of the month. These changes in the quantity of suspended matter below the reservoir apparently resulted from an increase in discharge and suspended matter observed at the Grand Canyon Station for a few days beginning September 27.

SIZES OF PARTICLES

Particular attention is called to the curve in Fig. 2 showing variations in sizes of particles, because it seems likely that the greater specific gravity due to the turbidity of the water, especially when the particles are very fine, may be an important factor in the movement of turbid water through the reservoir.

Fig. 2(*b*) shows the quantity of material with effective diameters of less than 20 microns that is carried daily past the Grand Canyon Station. Whenever there was an increase in the load of suspended matter at the Willow Beach Station there had been a prior increase in the quantity of material less than 20 microns in diameter at Grand Canyon. In each of the periods of turbid discharge, the suspended matter at Willow Beach consisted chiefly of small particles with slow rates of settling, more than 90% of the material being less than 20 microns in diameter.

There will naturally be speculation as to the reason why the high water of May and June, with its heavy silt load at Grand Canyon, did not produce a turbid discharge from Lake Mead. A possible explanation may lie in the fact that, at that time, a high percentage of the silt after dispersion (and probably the percentage was still higher before dispersion) was larger than 20 microns and that this large portion of the silt, with its fast settling rate, carried down with it practically all the finer silt. Determinations of size show that the material carried at Grand Canyon during this period was in a state of partial coagulation and that the sizes of the particles carried were larger than the analyses of the dispersed samples indicate.

During those parts of the period from February to August, when the loads at Willow Beach were small, the suspended material carried past this station consisted chiefly of particles, having diameters greater than 50 microns, picked up in the river channel between the dam and the station. During the same period a slightly larger load was carried past the Topock Station, but the sizes of the particles at the Topock Station were similar to those at the Willow Beach Station. Similar scouring of the river channel at certain periods has been indicated by the results obtained for several years before Boulder Dam was completed. During certain periods of the year (usually in the late fall and winter) before storage in Lake Mead was started, the load at the Topock Station was considerably greater than that at the Grand Canyon Station, and the differences could not be attributed to inflow from tributaries between the stations. It has seemed likely that at these times the river (which probably was not carrying a capacity load at the Grand Canyon Station) had eroded, from the bed and banks, material that had been previously deposited. It is significant that during a period of ten years the samples collected at Grand Canyon have often had less than 0.05% of suspended matter, whereas the samples collected at Topock have rarely had less than 0.10% of suspended matter. At such times of low percentage of suspended matter at the Topock Station, the load has consisted chiefly of particles more than 50 microns in diameter which would drop out of suspension rather easily.

Increases in discharge and suspended matter at the Grand Canyon Station, particularly in the late summer, are frequently caused by rains of high intensity in regions of little vegetation, resulting in rapid run-off of water, high in both dissolved and suspended matter. Herman Stabler,[11] M. Am. Soc. C. E., has presented data indicating that 60% of the suspended matter carried past the Grand Canyon Station came from 36% of the drainage basin lying above that point. The increase in discharge at Grand Canyon on August 26, 1935, was caused largely by an increase in the flow of the Little Colorado River, which drains a large arid region and enters the Colorado 26 miles above the Grand Canyon Station. The material carried by the Little Colorado during certain other summer floods has consisted chiefly of fine particles, and it appears likely that the Little Colorado brought into the Colorado a large part of the material carried past the Grand Canyon Gaging Station on August 26 and a few days subsequent. There is evidence that the Little Colorado and perhaps the San Juan may have brought to the Colorado much of the silt that appeared at Willow Beach during the three periods of turbid flow in 1935, but the evidence is not complete.

DISSOLVED MATTER

Changes in concentration of dissolved matter have been estimated approximately from the changes in sulfate, the principal acid radicle present in the Colorado River water. The determinations of sulfate have shown that a rapid increase in concentration of dissolved matter in the main river usually accompanies an increase in the quantity of suspended matter. The variations in sulfate at the Willow Beach Station, shown in Fig. 2(*a*), are of significance, therefore, in confirming the evidence afforded by the silt that, at times of silt discharge, water entering the reservoir is going through the reservoir without complete mixing. This evidence of absence of mixing is in agreement with the observations on the Rio Grande, where, it is reported by Fiock and Scofield[12] that the soluble salt content of the outflow increases materially during the periods of turbid flow. The simultaneous increases in sulfates and suspended matter at the Willow Beach Station would seem to indicate that they are definitely related to each other. The interval of about eight days between the increase at Grand Canyon and the increase at Willow Beach appears to be a reasonable time for water to flow the 275 miles from the Grand Canyon Station to the Willow Beach Station, including 70 to 90 miles through Lake Mead, with due allowance for the velocity of flow through the lake, resulting from an average slope of 3 or 4 ft to the mile of the bed of the reservoir, on which the turbid water probably flows beneath the quiescent clear water.

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[11] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 277.

SPECIFIC GRAVITY

Simple experiments can be made in the laboratory to show that water of greater specific gravity, whether such specific gravity is obtained by decrease in temperature, the solution of soluble material, or the suspension of solid material of such fineness that it will stay in suspension for several hours, will find its way to the bottom of a dish containing water of less specific gravity. If there is any flow of water through the dish, the water of greater specific gravity will move along the bottom to the outlet.

Causes for the differences in specific gravity should be considered on the assumption that the relatively rapid flow of unmixed water through a long reservoir is due to: Differences in specific gravity; the slopes of the surfaces of water of greater specific gravity; or the slope of the sediment on which the flow takes place. In the studies reported in "Hydraulic Laboratory Practice,"[3] the phenomena were discussed wholly on the basis of differences in temperature, although the differences in specific gravity of the water used in the laboratory experiments appear to have been obtained generally by adding a solution of salt and not by changing the temperature. As a result of those studies it was concluded that water of greater specific gravity due to lower temperature might flow essentially unmixed through a basin or reservoir containing water of less specific gravity, and that the depth at which the flow would take place would be determined by the specific gravity of the colder water. This same effect as related to temperature was discussed[13] by Armin Schoklitsch in 1922.

A new aspect, evidently not previously studied extensively and, perhaps, not pertinent in the foregoing studies,[3] is presented in connection with the Colorado River and doubtless many other silt-laden streams, by a water made heavy because of a load of fine silt that is not quickly dropped even when the velocity is reduced to zero. This fine silt is associated so closel with the water by which it is carried that it apparently increases the specifi gravity of the water. The range in specific gravity due to the presence o fine silt may be much greater than that due to differences in temperatur Under some conditions, therefore, water of an inflowing stream, even i it is higher in temperature, may not remain on the surface of a reservoi but may sink to the level determined by its greater specific gravity du to its load of fine silt. Mr. Fiock[6] has stated that the temperature of th outflow from the Elephant Butte Reservoir during periods when turbid wate is passing through the reservoir, not on its surface, is about 6° higher tha the temperature of the reservoir water. When the outflowing water become clear, its temperature drops to that of the reservoir water. Unfortunately there are no records of the temperature of Colorado River water as it enter into, or flows from, Lake Mead; nor of the water in Lake Mead at variou depths. No attempt has yet been made to find the depth of the flow o unmixed turbid water through Lake Mead, but it is tentatively assume that the flow of turbid water is just on top of previously deposited sil and that it follows the original channel of the river.

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[12] *Transactions*, Geophysical Union, 1934, Pt. 2, p. 472; also "The Movement of Silt through the Elephant Butte Reservoir", by C. S. Scofield. (Unpublished memorandum, February 17, 1936).

[13] "Einfluss der Durchleitung von Flusswasser durch Seen auf deren temperatur". Sonderheft to *Wasserkraft*, 17, 1922.

The differences between the specific gravity of the incoming water and of the lake water may be greater than the difference in specific gravity caused by differences in temperature. The specific gravity of pure water is 0.993 at 100° F, and has a maximum of 1.000 at 39.2° F. The specific gravity of the water that flowed through Lake Mead in October, 1935, ranged from 0.995 to 1.008, showing a greater variation than could be caused by the most extreme natural differences in temperature. Approximate values for the specific gravity of the turbid water that passed Lake Mead and the percentages of suspended matter in the water as determined on samples collected at the Willow Beach Gaging Station are given in Table 2.

TABLE 2.—SPECIFIC GRAVITY AND PERCENTAGE OF SUSPENDED MATTER IN WATER OF COLORADO RIVER AT WILLOW BEACH GAGING STATION, OCTOBER, 1935

Date (1)	Specific gravity* (2)	Suspended matter (percentage by weight) (3)	Date (1)	Specific gravity* (2)	Suspended matter (percentage by weight) (3)
October 5	0.995	0.004	October 10	1.008	1.76
October 6	1.005	1.38	October 11	1.007	1.76
October 7	1.002	1.32	October 12	1.004	1.14
October 8	1.004	1.40	October 13	0.999	0.38
October 9	1.003	1.00	October 14	0.996	0.01

* Determinations made at 84.2° F, referred to pure water at 39.2° F.

On the basis of present information it appears, therefore, that the phenomenon of flow of essentially unmixed turbid water through a long reservoir at one season or another is not uncommon, and that it may be affected by temperature, dissolved salts, size of particles of silt, and the degree of coagulation of dispersion of the silt particles.

The observation herein recorded relating to the flow of turbid water from Lake Mead during 1935 may have peculiar value, because they have been made systematically throughout the period when stored water was discharged through tunnels at, or near, the base of the dam, and, therefore, was drawn from the bottom of the lake. Early in 1936, while water was still being discharged through these tunnels, some of the water passing the dam was drawn through the intake towers from a level about 250 ft above the base of the dam. Under these conditions on April 22, 1936, the river at Willow Beach became turbid shortly after noon, the silt appearing in the main current at first and in 10 min the entire section was filled with turbid water. At this time, the discharge below the dam was about 10 000 cu ft per sec, of which about 5 000 cu ft per sec came through the diversion tunnel and about 5 000 cu ft per sec came through the outlet works. In this period of turbid flow about 2 400 000 tons of silt was carried past Willow Beach. The gate in the tunnel was closed on May 1, and the river at Willow Beach became clear on May 2.

CONCLUSIONS

The passage of silt in considerable quantities through a reservoir may have much more than scientific interest and importance. One of the great unsolved problems pertaining to the detention of water in reservoirs relates to the gradual loss of capacity caused by the deposition of silt. If any considerable quantity of silt can be carried through the reservoir its effective life will be correspondingly increased. In the three periods in 1935 of turbid flow from Lake Mead described herein 6 000 000 tons, or about 2.5% of the estimated average annual load of silt brought to the lake, has passed through it. The quantity of silt carried through other reservoirs on rivers having different silt and run-off characteristics has not been recorded and, perhaps, may reasonably be assumed to be smaller than that carried through Lake Mead. However, the quantity of silt thus carried may well be of sufficient importance to warrant further consideration.

The question may naturally be raised as to the possibility of increasing the discharge of silt from reservoirs created by future dams by modifying the positions of and approaches to the discharge openings or the manner of their operation. It is suggested that a study of the conditions pertaining to the flow of silt through a reservoir may be worth while. Engineers who make future designs will surely want to know the conditions that make such passage of turbid water possible, in order that full consideration may be given to any advantages that may be practicable of attainment by modifications in the details of design of the dams or in the plans for operation.

The record of silt flow described herein may be important not only to engineers in their evaluation of silt problems related to many other reservoirs built, or to be built, on silt-laden streams, but also in connection with certain geologic and engineering studies related to the deposition of silt and possibly to the origin of subaqueous channels. It is hoped that there may be brought into the discussion other definite information related to the little understood phenomena associated with the flow of water through reservoirs, and to the possible applications of the phenomena that may have practical significance with respect to engineering problems.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

PRACTICAL USE OF HORIZONTAL GEODETIC CONTROL

BY R. C. SHELDON,[1] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

An argument for the increased use of the horizontal geodetic control system, is offered in this paper. It contains: (1) An "Introduction," including suggestions to the engineer when he is beginning to use this control; (2) a description of the origin and use of such a system, followed successfully on the Panama Canal for twenty-four years; (3) an analysis of the errors developed by using the system of control in the Canal Zone; (4) an analysis of the errors developed by using the same system near the Fortieth Parallel of Latitude; and (5) the writer's conclusions.

INTRODUCTION

Recent Progress in Geodetic Control Work.—Since 1930 there has been a large increase in the activities of those organizations working on geodetic control in the United States. The main triangulation net of the United States Coast and Geodetic Survey is nearing completion, giving chains of triangulation stations spaced at not more than 25-mile intervals. In addition to this main net many intermediate triangulation chains and traverses have been established by various other organizations, and, more recently, by the State Relief Agencies. Some States have begun to tie their extensive highway surveys to the National Net.

All work already done, and much more that is planned for the near future, either has been, or is soon to be, reported on the basis of geodetic co-ordinates. It is time that engineers interested in surveying engage in some discussion to determine the best methods of using this control system. At present (1937), the only engineers who use the geodetic control, as it should be used, are those employed in the Mapping Divisions of the U. S. Coast and Geodetic Survey, the United States Geological Survey, the Hydrographic Office of the United States Navy Department, and other Federal agencies that produce maps covering large areas.

NOTE.—Discussion on this paper will be closed in August, 1937, *Proceedings*.

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Engineers Do not Use Geodetic Control.—The American engineer and surveyor refuses to work with geodetic positions. The published information on the subject has many statements explaining the need for Statewide plane co-ordinate systems, such as the following (11)[2]: "Of course, if all engineers would use geodetic computations, there would not be need for such aids, but they do not favor this method of handling the situation."

The reason for this reluctance to use geodetic control is apparent to any engineer who has attempted to adapt it to every-day survey work. If he finds a geodetic control point in the vicinity of his work he generally wants to use it. Then he goes through the following cycle: He looks up the position of the point in the geodetic reports and finds it given in degrees, minutes, seconds, and in decimal parts of seconds; in some cases the seconds are given in meters. In any case, he must go to the textbooks for information as to the number of meters in a second or in a minute of latitude and longitude, before he can base his survey on the point. There he finds: (*a*) Pages of material on the methods used in reconnaissance for the triangulation; (*b*) more pages on the methods of observation; (*c*) lengthy descriptions on the measurement and accuracy of base lines; (*d*) some sketchy information as to the computations; (*e*) detailed description of the relatively simple operation of precise leveling; and (*f*) a reference to the Government publication from which the information given, was taken. Finally, he abandons the system, reasoning that if the computation of seconds to meters is too complicated for the author of a textbook on surveying, it is too complicated to use.

The publications of the Government organizations covering the subject (see Appendix), if taken as a whole and carefully studied, give a complete picture of the necessary process; but, unless one has all of them at his disposal and studies them thoroughly, he is likely to become confused and decide that geodetic control is to be used for surveys covering large areas only. The beginner will not find a clear step-by-step explanation as to how to tie a local survey to geodetic co-ordinates, except by the use of plane co-ordinates.

Plane Co-Ordinates Used as Substitute.—Realizing the necessity for tying their work to the national net, some of the more progressive cities have made geodetic control surveys of their area; but as a basis for actual work they have substituted plane co-ordinates, which are about to be adopted as a standard practice. This is unfortunate because the objections to the use of plant co-ordinates are numerous, and increase with the distance from the origin. In addition to the fact that it is fundamentally wrong to use plane co-ordinates in surveying areas on the spherical surface of the earth as if it were a plane, they are also confusing to the surveyor who has always used the meridian as his line of reference. True azimuths must be corrected when using plane co-ordinates, the correction depending on the position of the point with reference to the origin of co-ordinates. Errors are certain to occur where two co-ordinate systems join.

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[2] For reference to figures in parentheses, see "Bibliography", in Appendix.

SIMPLICITY OF HORIZONTAL GEODETIC CONTROL MUST BE EMPHASIZED

In order that the engineer may be encouraged to use this geodetic control in his every-day work (and it would be an aid to every engineer having any kind of survey work to do), its simplicity must be emphasized. A comparable example is in the use of geodetic levels. Their advantages are instantly admitted and they are used whenever possible, even where profile levels must be run for miles to base some project on the common sea-level datum. If the horizontal geodetic system were made as simple to understand as the vertical system, it would be used by all engineers.

Horizontal Geodetic Control Is Easy to Understand.—Surveyors and engineers have generally understood and used the fundamental principles of geodesy; that is, that the earth is an approximate sphere, that north is always to the pole, that the meridians of longitude converge, and that parallels of latitude are curved lines when run on the earth's surface. Children are taught in the elementary grade schools that the earth is a sphere and that points are located on it by measuring from the equator and the meridian through Greenwich, in terms of degrees and minutes of latitude and longitude. Therefore, it is easy for any layman to understand that the corners of his lot or farm are at a certain latitude and longitude, based on these well known references.

When engineering students are taught in college that they are to ignore all that they have previously learned about the shape of the earth; and that they are to consider it a plane surface for surveying purposes, because the solutions of geodetic problems are too difficult, they are led a step backward; a step that is entirely unnecessary. They should be told instead that: (1) All points located by the U. S. Coast and Geodetic Survey are tied to the equator and to the Greenwich meridians by an average of many observations; (2) it is their duty as surveyors to tie their surveys to such points, wherever possible; and (3) the computations to do this are no more difficult than those involved in ordinary surveying.

Information to Beginners.—For those who have never made any study of geodetic control methods the following simple facts will be an aid.

The computation of geodetic positions is a distinct and separate operation from adjustment of any kind; that is, no least squares computations are involved in the simple computation of geodetic positions. Having the geodetic position of one point and the geodetic azimuth and the distance from that point to another, the geodetic position of the second point and the geodetic back azimuth are computed in the standard printed forms.

The computations for difference in latitude are made by multiplying the length of the line between the points by the cosine of the azimuth times a factor taken from tables. The purpose of this factor is to convert the meters of length into seconds of arc, at the latitude of the initial station. This latitude difference is then corrected by three other values all of which vary, in some degree, with the sine of the azimuth (that is, they depend on the difference in longitude). The computation for difference in longitude is made by multiplying the length of the line by the sine of the azimuth times the secant of the latitude, and by a factor, taken from tables at the latitude of the new station. This factor converts the meters into seconds. No corrections are made to the difference in longitude.

Although the standard forms for geodetic computations of both the Coast and Geodetic Surveys and the Geological Survey are well arranged for the work as done by those Bureaus, they are not a good guide to the beginner. The computations are done in steps as follows: (*a*) Having the geodetic azimuth from one known point to another known point by a previous computation, the geodetic azimuth to the unknown point is computed by a simple addition or subtraction of the angles at the known point (spherical excess corrections are necessary in the case of very large triangles); (*b*) the difference in latitude is then computed by the steps and corrections shown on the standard form, and is followed by the computation for the difference in longitude; and (*c*) the difference in azimuth is computed last. On short lines it is not necessary to apply all the corrections to the difference in latitude as the values found will be negligible. The entire operation requires only a knowledge of logarithms and simple trigonometry.

Astronomic azimuths and positions are those that are determined by observations on the sun, moon, or stars. Geodetic azimuths and positions are those that are computed and carried forward by geodetic computations. Astronomic azimuths and positions should check with the geodetic azimuths and positions, except for small corrections.

CONTROL SYSTEM OF PANAMA CANAL ZONE

Origin.—Since 1911, the Section of Surveys of the Panama Canal Zone has used a system of geodetic control as the base for all surveys. It has been found to be of great value in avoiding confusion, as all surveys are reported with positions of latitude and longitude. No special study is necessary by those using the system. In the vicinity of the Canal surveys are made by five different Canal or other Government organizations as well as by some private engineers. It is possible for each of these agencies to use the work done by any or all the others because all surveys are tied to the one common Panama-Colon Datum.

At the beginning of the construction of the American Canal, and for the several years of the preliminary work, a situation existed on the Isthmus, with respect to survey control, that was much worse than that in the United States prior to the establishment of the Standard North American Datum. There were many different systems of surveys that joined and overlapped each other without a common base. On the Atlantic side of the Isthmus there was a base line and a triangulation system which were based on an astronomical determination of a point on the Colon Light-House. On the Pacific side, there was another system of triangulation based on an astronomical determination of the south tower of the Panama Cathedral. The two astronomical determinations were originally made by outside departments for coast surveys. There was no definite cross-tie between the two systems, although there were many survey lines covering the construction area, established by both the French and the American engineers. This situation led to much confusion especially where the work of two separate divisions met.

In 1910 and 1911 a base line, 3 785.454 m long, was surveyed on the Tavernilla Dump (now covered by Gatun Lake) and measured to a probable error of 1:700 000. A triangulation system, called the primary system for local purposes, consisting of twenty-three stations (including the ends of the base line and the stations of the old Atlantic and Pacific Systems) was carefully located and measured. The triangles of the primary system had an average closure of 1.98″ and a maximum closure of 5.12″. The computations and adjustments were made in Washington, D. C., by the U. S. Coast and Geodetic Survey, by which it was classed as secondary work. Serious discrepancies were found between the positions of the two systems on the opposite sides of the Isthmus. After having made all possible corrections to these positions, dual positions were computed through the primary system (using the initial astronomic value of each of the old systems) for the Balboa Station in the center of the Canal Zone. These two positions were then averaged and the mean position of the central point was adopted as the common datum, called the Panama-Colon Datum.

Approximately seventy supplementary triangulation points were established, and the angles observed, computed, and tied to the primary system; these angles were observed with lesser care than those of the primary system and the computations were made locally; they are considered the secondary system. These triangulation stations were the base for all surveying done on the construction after 1911 and since, in the maintenance and operation of the Panama Canal. They are also used for the control of that part of the military maps covering the vicinity of the canal, and as a base for extending triangulation surveys covering both coasts. Since the establishment of the main control system, it has been necessary to extend it for particular points, and to produce control to desired places. There are now (1937) about 250 triangulation points (most of which are second-order points) in the area around the Canal Zone; the controlled area averages about 4 sq miles per station. This complete system is comparable to the geodetic control net of the United States and all the supplementary work that has been done to extend it.

When this triangulation system was established it was intended to be used as a control on which a rectangular system of surveys would be based. The proposed plan was similar to that of the General Land Office surveys (although not in method), modified somewhat to resemble the systems of rectangular co-ordinates that were then being adapted to some of the large cities in the United States. Monuments for the 2-km intersections of the co-ordinate lines were to be located in the field. This plan failed when it was less than one-half completed, mostly because of the impossibility of locating or using the intersection points on the rough topography of the Canal Zone. Shortly afterward, it became necessary to do considerable surveying to settle land claims and to locate some of the topo- graphical features in the outlying parts of the Zone. To complete this work the surveyors, some of whom had previously worked on the triangulation system and were thus familiar with geodesy, adopted the present system as a practical substitute for the planned scheme.

Control of Maps and Municipal Surveys.—In working from the triangulation net in the Canal Zone the procedure varies with the class of surveying to be done. For most of the surveys in the occupied parts of the Canal Zone, at the terminals, and along the Panama Railroad, the Section of Surveys has placed control points on the curbs of the streets and in other accessible places. These points are located by ordinary transit and chain traverse with an accuracy of 1:5 000. Since there is no private ownership of property on the Canal Zone, there is no use in maintaining a higher accuracy than is necessary for the control of ordinary municipal work in a suburban district of an American city.

For work involving the boundaries between the Republic of Panama and the Canal Zone, or for the boundaries of the military and naval reservations, transit traverses are tied directly to the triangulation system. Such surveys are run with a varying accuracy, depending on the location and importance of the lines. For the computations of positions on all the transit traverse lines the procedure is as follows:

(1) All triangulation stations are recorded in a card index. The card for each station gives the position of the station and the azimuths and distances to the observed stations (see Table 1).

TABLE 1.—SAMPLE CARD FROM FILE OF TRIANGULATION STATIONS
(Ancon; Primary; Elevation, 654.0 Feet)

Latitude	**8° 57′ 25.526″**	**784.2 m**	**2572.75 ft**
Longitude	**79° 33′ 08.813″**	**269.2 m**	**883.26 ft**
	Azimuth:	**Back Azimuth:**	**Station:**
	125° 51′ 23.51″	**305° 50′ 18.62″**	**Gordo**
	170° 05′ 04.07″	**350° 04′ 51.19″**	**Iron**
	Distance, in Meters	**Log Distance:**	**Station:**
	15 627.73	**4.1938958**	**Gordo**
	14 630.51	**4.1652595**	**Iron**

The positions are given with the seconds reduced to feet; this reduction is made very easily from tables which give the length, in feet, to three decimal places for the seconds of both latitude and longitude for every minute of latitude. As the variation between minutes is small, interpolation between minutes is satisfactory for most purposes. If a more accurate determination is required, the formula for any latitude is taken from *Special Publication No. 5* of the U. S. Coast and Geodetic Survey (6).

The surveyor consults this card index before beginning a survey from a station and uses the correct azimuth. On closing his survey on another station his azimuth will be in error (in addition to the errors made in unning the line) an angle equal to the difference in geodetic azimuth ecause of the difference in longitude of the two stations. No lines are un locally that are closed on the same station; that is, closed circuits. n computing the traverse this error, with all the field azimuth errors, is alanced evenly over the angles to close on the two triangulation stations. Thus, the azimuths in the computations are approximately correct, depending on the evenness of the course lengths; if some lines are much shorter than others, or if a series of short lines is followed by a series of long lines, errors will be introduced. For most of the work the errors in running the line are far greater than those that develop from this difference in azimuth between stations. It is scarcely realized by the men doing the work that there is any difference.

After the angles are balanced the line is then computed in the ordinary manner by converting the azimuths into bearings and using the sine and cosine to obtain the latitudes and departures. The line is then adjusted to close on the triangulation stations by distributing the error of closure by any method that is justified by the accuracy of the field work.

TABLE 2.—FEET PER SECOND AND PER MINUTE OF LATITUDE AND LONGITUDE

Latitude	LATITUDE		LONGITUDE	
	Feet in 1′	Feet in 1″	Feet in 1′	Feet in 1″
(a) IN THE PANAMA CANAL ZONE; LATITUDE, 9° N				
8° 56′	6 047.36	100.789	6 013.73	100.229
8° 57′	6 047.36	100.789	6 013.46	100.224
8° 58′	6 047.37	100.789	6 013.18	100.220
8° 59′	6 047.37	100.789	6 012.91	100.215
9° 00′	6 047.38	100.790	6 012.63	100.211
9° 01′	6 047.38	100.790	6 012.36	100.206
9° 02′	6 047.39	100.790	6 012.08	100.201
9° 03′	6 047.39	100.790	6 011.81	100.197
9° 04′	6 047.40	100.790	6 011.53	100.192
9° 05′	6 047.41	100.790	6 011.25	100.188

Latitude	LATITUDE		LONGITUDE	
	Feet in 1′	Feet in 1″	Feet in 1′	Feet in 1″
(b) IN THE UNITED STATES; LATITUDE, 40° N				
40° 00′	6 071.33	101.189	4 669.51	77.825
40° 01′	6 071.34	101.189	4 668.37	77.806
40° 02′	6 071.36	101.189	4 667.24	77.787
40° 03′	6 071.38	101.190	4 666.10	77.768
40° 04′	6 071.40	101.190	4 664.96	77.749
40° 05′	6 071.41	101.190	4 663.83	77.731
40° 06′	6 071.43	101.191	4 662.69	77.712
40° 07′	6 071.45	101.191	4 661.55	77.693
40° 08′	6 071.47	101.191	4 660.43	77.694
40° 09′	6 071.49	101.192	4 659.28	77.655

For the computation of lines using this system it is necessary to have a table giving the value, in feet, to the nearest hundredths of feet, of the minutes of latitude and longitude, for each minute of latitude. Such a table, a section of which is shown in Table 2, covering the two degrees of latitude of local interest, consists of one sheet slightly larger than letter size; the table, mounted on cardboard, is easily consulted by computers. When, in summing up the feet of latitude or longitude, the values go above those for 1′, the proper value taken from Table 2(*a*) is subtracted or added to the sums and the next minute placed in the minute column.

Table 2(*a*) can be prepared in several ways. The method that gives the more accurate value in feet is, to make an inverse geodetic calculation to obtain the distance, in meters, between two points on the meridian or on the parallel 60″ apart, at the latitude desired. In the case of a line with an azimuth of 0°, or 180°, the computation for obtaining the distance in latitude reduces to the simple division of the distance between the stations, in seconds, by the factor taken from the tables, as all the corrections contain multiple of the sine of the azimuth, which is zero in this case, and the other term is divided by the cosine which is unity. The value found by the inverse calculation is in meters and is converted to feet to be included in Table 2(*a*). The computation for the distance in longitude is made in the same manner; the operation of making the inverse calculation requires only one multiplication and one division in this case, in addition to the reduction of meters to feet. These two tables (Tables 2(*a*) and 2 (*b*)) can also be prepared by the use of tables that can be found in *Special Publication No. 5* (6), which give the lengths of the seconds of both latitude and longitude for each minute of latitude in meters. This latter method does not give accurate values in the last decimal place because of a lack of sufficient significant figures in the values given in *Special Publication No. 5* of the U. S. Coast and Geodetic Survey (6).

Methods Used for More Accurate Surveys.—Triangulation is used for locating points with a higher degree of accuracy than by the transit traverse method, or for determining the positions in a truly geodetic manner. Quadrilaterals, using the stations of the existing system, are observed and adjusted. The observations are made with a repeating theodolite, reading to 10″, six complete sets of angles, both direct and reversed, and horizon closures, are turned. The standards of secondary triangulation are obtained without difficulty. The geodetic positions are computed by the use of a standard form (No. 9–902) prepared by the U. S. Geological Survey (4). Values for the corrections can be found in *Bulletin 650* (4), or in *Special Publication No. 8* (5).

ANALYSIS OF ERRORS

Errors in the Computations of Traverses.—The foregoing system of computation for traverse positions does not give exact geodetic positions, but it does give positions that are nearer to the truly geodetic than can be determined with the survey methods used. The line that would yield the nearest to the true geodetic positions of the points would be one with many equal courses. The balanced azimuths would then make the line follow the curve of the earth and the computed positions, to be nearly geodetic. The line creating the worst condition would be that containing one long straight course. The direction of the line also affects the difference between the positions as computed by this system and the true geodetic positions. Lines running north or south are affected very little by the geodetic corrections, being only concerned with the difference between the spheroid of reference and a true sphere, whereas lines running east or west must be corrected for convergence of meridians. For these reasons the computations for errors were made on the basis of a line running due west consisting of one single course.

Table 3(*a*) was prepared by taking an even degree of latitude and longitude as a base and computing geodetic positions of points at varying distances on a straight line beginning at the base, with an azimuth of 90 degrees. These positions are placed in Columns (2) and (4). They would represent triangulation stations at the various distances apart. The difference in azimuth is placed in Column (6), Table 3(*a*), and shows the maximum difference that would appear in the azimuths of lines run between stations at the same distances apart as those in Column (1). The positions of the same points on the same straight line are then computed by the use of the sine, cosine, and table method (see Column (7)). The difference between these positions and the true geodetic positions is shown in Column (8), Table 3(*a*). It will be noted that the errors in longitude are very small. The proportional error is shown in Column (9).

TABLE 3.—MAXIMUM ERROR POSSIBLE

Distance, in thousands of feet (1)	GEODETIC POSITION				Azimuth difference (6)	Longitude by sine and table* (seconds are expressed in feet) (7)	Latitude error, in feet (8)	Proportional error, one in: (9)
	Latitude 8° 59″ plus seconds: (2)	Seconds of latitude, in feet (3)	Longitude (4)	Seconds of longitude, in feet (5)				
			(*a*) IN THE PANAMA CANAL ZONE; LATITUDE 9° N.					
0......	00.00000″†	6 047.38	79° 00′ 00.0000″	0.00	000	79° 00′ 0000.00	— 0.00	0
1......	59.99996″	6 047.376	79° 00′ 9.9790″	1 000.00	0′ 1.56″	79° 00′ 1000.00	— 0.004	250 000
5......	59.9991 ″	6 047.29	79° 00′ 49.8950″	5 000.00	0′ 7.80″	79° 00′ 5000.00	— 0.09	56 000
10......	59.9962 ″	6 047.00	79° 01′ 39.7898″	3 987.37	0′ 15.61″	79° 01′ 3987.37	— 0.38	26 000
20......	59.9850 ″	6 045.87	79° 03′ 19.5798″	1 962.11	0′ 31.22″	79° 03′ 1962.11	— 1.51	13 000
30......	59.9629 ″	6 043.64	79° 04′ 59.3697″	5 949.50	0′ 49.04″	79° 04′ 5949.50	— 3.74	8 000
50......	59.9061 ″	6 037.92	79° 08′ 18.9495″	1 898.95	1′ 18.05″	79° 08′ 1898.96	— 9.46	5 200
100......	59.6246 ″	6 009.54	79° 16′ 37.899 ″	3 797.90	2′ 36.11″	79° 16′ 3797.92	—37.84	2 600
			(*b*) IN THE UNITED STATES: LATITUDE, 40° N.					
0......	00.00000″‡	6 071.33	79° 00′ 00.0000″	....	....	79° 00′ 0000.00¶	0.00	0
1......	59.9998″§	6 071.31	79° 00′ 12.8493″	1 000.00	0′ 8.26″	79° 00′ 1000.00¶	0.02	50 000
5......	59.9951″§	6 070.83	79° 01′ 04.2466″	330.49	0′ 41.30″	79° 01′ 330.49¶	0.50	10 000
10......	59.9802″§	6 069.33	79° 02′ 08.4934″	661.00	1′ 22.59″	79° 02′ 660.98¶	2.00	5 000
20......	59.9209″§	6 063.32	79° 04′ 16.9865″	1 321.97	2′ 45.19″	79° 04′ 1321.96¶	8.01	2 000
30......	59.8048″§	6 051.57	79° 06′ 25.4794″	1 982.93	4′ 7.78″	79° 06′ 1982.94¶	19.76	1 500
50......	59.5053″§	6 021.28	79° 10′ 42.4663″	3 304.94	6′ 52.97″	79° 10′ 3304.90¶	50.05	1 000
100......	58.0224″§	5 871.22	79″ 21′ 24.933 ″	1 940.41	13′ 45.94″	79° 21′ 1940.29¶	200.00	500

* Latitude, by cosine and table, equals 9° 00′ 00″. † 9° 0′: ‡ 40° 0′: § 39° 59′: ¶ Latitude by cosine and table, equals 40° 00′ 00″.

Table 3(*a*) shows the maximum errors that will be introduced in any line run between triangulation stations which are the distances apart shown in Column (1). It is clear that this maximum error will not be reached in practice as it would require running one straight course between triangulation stations. If this were done the balanced azimuths would be the average of the forth and back azimuths of the line and, if this were used in the sine, cosine, and table method, the line would check exactly. To show a more practical determination of the errors it was assumed that the ends of the aforementioned line were to be triangulation stations 100 000 ft apart, and that a traverse was run between them with points as shown in Column (1), Table 4(*a*). The angles were then balanced over these courses and the traverse was computed in the usual way, using the sine, cosine, and Table 2. The unbalanced traverse is compared with the true geodetic positions and these errors are shown in Columns (7) and (8), Table 4(*a*). The line was then corrected to close on the triangulation point by a simple distribution of the error of closur according to the lengths of the courses, thereby lessening the errors to th values, as shown in Columns (10) and (11). Table 4(*a*) shows the error in this particular traverse; it would be possible to arrange a traverse tha would give any closures and errors desired, but this was considered to b an example of the worst condition ever likely to be encountered in the field Most of the traverses run will have a better distribution of length than this, and will have errors that will be much less than those given i Table 4(*a*).

TABLE 4.—ERRORS ON EXTREME EXAMPLE

Station (1)	Distance, in thousands of feet (2)	Balanced azimuth*; 89°: (3)	Increment of latitude, in feet (4)	Position latitude (seconds in feet); 8° 59′: (5)	Geodetic position (seconds in feet); 8° 59′: (6)	Error, in feet (7)	Proportional error, one in: (8)	Balance to close, in feet (9)	Final error, in feet (10)	Proportional error, one in: (11)
(*a*) IN THE PANAMA CANAL ZONE; LATITUDE 9° N										
0.....	0	....	....	....	6 047.38	00.00	......	....	00.00	......
1.....	1	59′ 40.5″	− 0.09	+6 047.29	6 047.38	− 0.09	11 000	+ 0.17	+ 0.08	12 000
5.....	4	59′ 21.0″	− 0.76	+6 046.53	6 047.29	− 0.76	6 400	+ 0.87	+ 0.11	45 000
10.....	5	59′ 01.5″	− 1.44	+6 045.09	6 047.00	− 1.91	5 200	+ 1.74	− 0.17	53 000
20.....	10	58′ 42.0″	− 3.78	+6 041.31	6 045.87	− 4.56	4 400	+ 3.48	− 1.08	18 000
30.....	10	58′ 22.5″	− 4.73	+6 036.58	6 043.64	− 7.06	4 200	+ 5.22	− 1.84	16 000
50.....	20	58′ 03.0″	− 11.37	+6 025.21	6 037.92	−12.71	3 900	+ 8.70	− 4.01	12 500
100.....	50	57′ 43.5″	− 33.08	+5 992.13	6 009.54	−17.41	5 700	....	....	......
(*b*) IN THE UNITED STATES; LATITUDE 40° N										
0.....	0	....	....	+0 000.00†	0 000.00†	....	......	....	....	......
1.....	1	58′ 17″	− 0.50	+6 070.83‡	6 071.31‡	− 0.48	2 100	+ 0.92	+ 0.44	2 300
5.....	4	56′ 34″	− 3.99	+6 066.84‡	6 070.83‡	− 3.99	1 300	+ 4.61	+ 0.62	8 000
10.....	5	54′ 50″	− 7.51	+6 059.33‡	6 069.33‡	−10.00	1 000	+ 9.22	− 0.78	13 000
20.....	10	53′ 07″	− 20.02	+6 039.31‡	6 063.32‡	−24.01	830	+18.44	− 5.57	3 600
30.....	10	51′ 24″	− 25.01	+6 014.30‡	6 051.57‡	−37.27	800	+27.66	− 9.61	3 100
50.....	20	49′ 41″	− 60.02	+5 954.28‡	6 021.28‡	−67.00	750	+46.10	−20.90	2 400
100.....	50	47′ 57″	−175.26	+5 779.02‡	5 871.22‡	−92.20	1 100	....	....	......

* Original azimuth, 90° 00′ 00″. † 40° 00′ plus. ‡ 39° 59″ plus.

Use of This Same System in the United States.—The personnel of the Section of Surveys is hired locally or is requisitioned from the United States from young men with some college training and with general field experience. No attempt is made to find men trained for geodetic work. These men work satisfactorily with this system after a short period of instruction. They soon become familiar with the common geodetic operations and understand the principles on which their surveys are based. The superiority of the system is such that these new men soon wonder why a similar system is not used generally in the United States.

One of the reasons for the adoption and the success of this system of surveys on the Canal Zone is the fact that, being near the equator (the Canal Zone is between Latitudes 8° 55′ and 9° 25′), the effects of geodetic corrections are much less than they are 30° or 40° farther north. Tables 2(*b*), 3(*b*), and 4(*b*), similar to Tables 2(*a*), 3(*a*), and 4(*a*) for the Ninth Parallel of Latitude, were prepared to show the errors that would be develope in using the same system at the Fortieth Parallel of Latitude.

CONCLUSIONS

The conclusions that can be made from the arguments given in this paper are:

(1) Plane surveying is out of date, and any mention of it should be avoided. Any system of surveying that does not fit in with the positions of the National Geodetic Net is doomed to eventual failure.

(2) If it were clearly understood by the engineers of the United States that the computation of geodetic positions had nothing to do with least squares adjustments of any kind, and that these computations were very easy to make, geodetic control would come into common use.

(3) A system such as that described herein is a valuable aid in performing all kinds of field engineering work.

(4) For the greater part of engineering work the lines can be run between the triangulation stations, ignoring the geodetic corrections, because the errors of the measurements in the field are more than those introduced by ignoring the geodetic corrections.

(5) For the few occasions when it is necessary to determine positions that require the use of trangulation and the computation of geodetic positions these positions can be determined by the average engineering organization without special training or instruction.

ACKNOWLEDGMENTS

The Canal Zone system of triangulation was begun under the direction of C. M. Saville, M. Am. Soc. C. E., then Assistant Engineer in charge of the Third Division of the Office of the Chief Engineer of the Isthmian Canal Commission. The method of using the system was begun under his direction and has been continued by the Section of Surveys of The Panama Canal, under the direction of Mr. O. E. Malsbury, from 1913 to 1926, and Mr. R. Z. Kirkpatrick, from 1926 to 1936, as Chiefs of Surveys. Various extensions of triangulation and improvements of tables were made by the Fortifications Division of The Panama Canal, under the direction of Assistant Engineer R. C. Jones, and by the Dredging Division of the Panama Canal, under the direction of the Superintendent, J. G. Claybourn, M. Am. Soc. C. E.

APPENDIX

BIBLIOGRAPHY

The books and publications that were reviewed in the preparation of this paper, and from which information was taken, are:

(1) **The Theory and Practice of Surveying,** by the late J. B. Johnson, M. Am. Soc. C. E., and Leonard S. Smith, M. Am. Soc. C. E.

(2) **Surveying Theory and Practice,** by Raymond E. Davis, M. Am. Soc. C. E., and Francis S. Foote, and W. H. Rayner, Assoc. Members, Am. Soc. C. E.

(3) **Topographic Instructions of the United States Geological Survey,** by C. H. Birdseye, M. Am. Soc. C. E., *Bulletin 788,* U. S. Geological Survey.

(4) **Geographic Tables and Formulas,** compiled by Samuel S. Gannett, *Bulletin 650,* U. S. Geological Survey.

(5) **Formulae and Tables for the Computation of Geodetic Positions,** *Special Publication No. 8,* U. S. Coast and Geodetic Survey.

(6) **Tables for a Polyconic Projection of Maps and Lengths of Terrestrial Arcs of Meridain and Parallel,** *Special Publication No. 5,* U. S. Coast and Geodetic Survey.

(7) **General Instructions for the Field Work of the United States Coast and Geodetic Survey,** *Special Publication No. 26,* U. S. Coast and Geodetic Survey.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

PRESSURES BENEATH A SPREAD FOUNDATION

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SYNOPSIS

An attempt has been made to confine the scope of this paper as nearly as possible to the limits defined by the title. Hence, discussion of tangential stresses (shears) and of pile foundations has been omitted entirely. In the "Introduction" an outline of stress distribution formulas is given, with special emphasis on the use of the "concentration factor," a conception applied to soil mechanics only since 1934.

Part I contains a description of a method of determining pressures under both uniformly and non-uniformly loaded foundations, except the case in which unit pressures on a part of the loaded area are infinite. This graphical process, termed the "reduced area method," may be applied in the case of a structure of arbitrary shape, and of an earth mass with an arbitrary concentration factor. Part II deals with the rigidity of structures and earth masses principally from the standpoint of their interaction.

Each of the two parts of the paper can be read independently. Only elementary mathematics, including trigonometry, has been used, the proofs of statements that require the application of calculus being presented in Appendices I and II.

Notation.—The letter symbols introduced in this paper are assembled for convenience of reference in Appendix III.

INTRODUCTION

Every theory of stress distribution within a body applies necessarily to an ideal material. Earth is not such a material, and theoretical formulas for semi-infinite isotropic bodies as used in this paper, may furnish data for an estimation of stresses within an earth mass, only if the latter is more or less homogeneous and isotropic to a considerable depth. Since stresses within an earth mass cannot be computed accurately, but can only be estimated, quick graphical methods are advanced in this paper to replace cumbersome analytical procedures.

NOTE.—Discussion on this paper will be closed in **August, 1937,** *Proceedings.*

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Stress Distribution Formulas.—Consider the infinitely large mass in Fig. 1. A concentrated vertical force, P, is applied to its horizontal surface at Point A, thus creating unit stresses, s, within the mass, which decrease in intensity at a rate proportional to the square of the distance from the origin (Point A). At any point of the mass (O, in Fig. 1), the unit stress produced by the force, P, at Point A, will be:

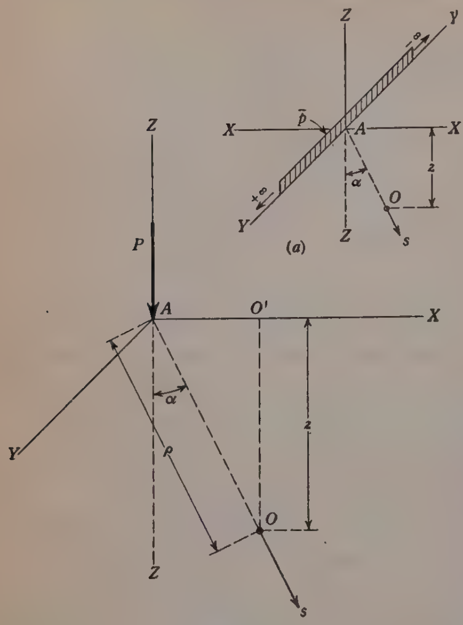

FIG. 1.

$$s = \frac{n\,P}{2\,\pi\,\rho^2}\cos^{n-2}\alpha \;\ldots\ldots(1)$$

Equation (1) has been proposed by John H. Griffith, M. Am. Soc. C. E.[2], and Dr. O. K. Fröhlich,[3] of Holland. Full credit for the development of a comprehensive theory of the concentration factor should be given to Dr. Fröhlich. The "concentration factor," n, is somewhat greater than 3 in the case of clays, and 6 or 7 in the case of coarse sands. It equals exactly 3 in the ideal case in which the mass under consideration follows Hooke's law, and is isotropic and incompressible, so that it may change its shape but not its volume. In such a case:

$$s = \frac{3\,P}{2\,\pi\,\rho^2}\cos\alpha \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Equation (2) may be obtained from the formulas developed by Boussinesq.[4] If the mass is loaded at its horizontal surface with an infinitely long unit load, expressed as $\overline{p}$ units of weight per unit of length, and acting along the axis, Y-Y (Fig. 1(a)), the stress distribution in the plane, XAZ, and in every vertical plane parallel to it, will be the same. The stress, s, at Point O, will be:

$$s = \frac{2\,\overline{p}}{\pi\,\rho}\cos\alpha \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Equation (3) represents the two-dimensional (plane) analogue of Equation (2). This is the so-called "Michell's radial distribution."[5] It shows that, in the case of a plane stress distribution within an elastic isotropic body, the stress intensities are inversely proportional to the distances from the origin of stresses, *A*.

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[2] "Pressures under Substructures", *Engineering and Contracting*, March, 1929, pp. 113–119.

[3] "Druckverteilung im Baugrunde", 1934; also, *De Ingenieur*, April 15, 1932.

[4] "Application des Potentials a l'Etude de l'Equilibre et du Movement des Solides Elastiques", Paris, 1885; also, Progress Report of the Special Committee on Earths and Foundations, Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., May, 1933, p. 777.

Since α is the angle formed by the stress, with the vertical, the vertical pressure, p_z, at Point *O* (Fig. 1), is:

$$p_z = s \cos^2 \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

To find the two-dimensional analogue of the general formula, Equation (1), the vertical pressure at Point *O* caused by the action of a straight line, (Axis *Y-Y*, Fig. 1), with a load, $\bar{p}$, per unit of length, bearing on the boundary, must be considered. A proposed method is developed in Appendix I. The value of the vertical pressure at Point *O* (Fig. 1) would be:

$$p_z = \frac{n_1 \, \bar{p}}{\pi \, \rho} \cos^n \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

From Equation (5) the value of the plane stress in a general case is:

$$s = \frac{n_1 \, \bar{p}}{\pi \, \rho} \cos^{n-2} \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

For the sake of brevity and for the purposes of this paper only, the coefficient, $n_1 = f(n)$, may be termed the "plane concentration factor," with values corresponding to n, as shown in Table 1. When $n = 3$, the value of n_1 equals 2, and Equation (6) becomes identical with Equation (3). Equations (1) and (6) can be applied in the case of any mass regardless of whether or not it follows Hooke's law. Since in this paper stresses are not determined from strains, the resultant of stresses coming to a point from two or more different sources may be determined by superposition.

TABLE 1.—VALUES OF CONCENTRATION FACTORS

Values of the concentration factor, n	$\frac{n}{2\,\pi}$	Corresponding values of the plane concentration factor, n_1	$\frac{n_1}{\pi}$	Values of the concentration factor, n	$\frac{n}{2\,\pi}$	Corresponding values of the plane concentration factor, n_1	$\frac{n_1}{\pi}$
3..........	0.477	2.00	0.637	6..........	0.955	2.95	0.939
4..........	0.637	2.36	0.751	7..........	1.114	3.20	1.019
5..........	0.796	2.67	0.850	8..........	1.27	3.44	1.090

PART I.—THE "REDUCED AREA" METHOD OF DETERMINING PRESSURES UNDER STRUCTURES

Pressures under uniformly loaded structures (vertical unit load, p) will be first determined; and for the purposes of Part I it will be assumed that the unit soil reaction, p, is also uniformly distributed throughout the foundation, and that it is vertical. Non-uniform loading, except the case of infinite unit loads, will also be discussed; and the corresponding soil reaction at a point of the earth's surface will be assumed vertical and equal to the unit load at that point.

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[5] *Proceedings*, London Math. Soc., Vol. 32, p. 35 (1900); also, "Theory of Elasticity", by S. Timoshenko, p. 82 (1934).

THREE-DIMENSIONAL PROBLEM

Referring to Fig. 2, the problem is to determine the vertical pressure, p_z, at Point O. The horizontal projection, O', of this point on the boundary plane is selected as a center, and the loaded area, M' N', is subdivided into zones by concentric circles, as shown in the plan view. The radii of these circles, although arbitrary, should be chosen so as to make the division curves pass through marked breaks in the perimeter of the loaded

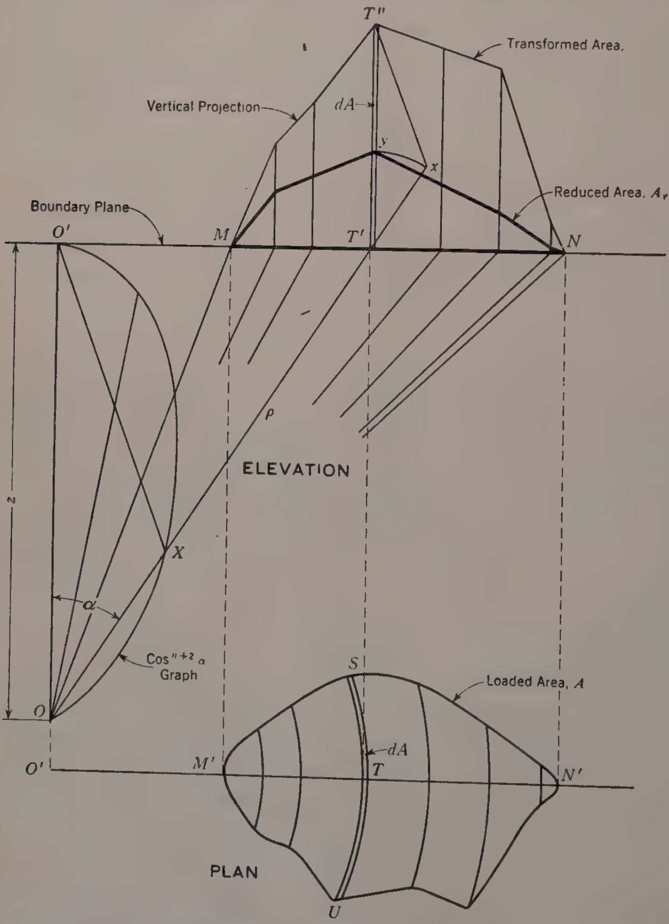

FIG. 2.—VERTICAL PRESSURE AT A POINT DETERMINED BY THE REDUCED AREA METHOD (THE THREE-DIMENSIONAL PROBLEM).

areas. Credit should be given to Messrs. Kögler and Scheidig[6] who first proposed subdividing a loaded area into circular rings, and not into rectangles, as is usually the practice. The area of an elementary zone, dA, practically equals the length of the curve, STU, times the width of that zone. The length of the curves is measured simply by "stepping off" small openings of a compass. Plotting the vertical ordinates, $T'T'' = STU$, a figure, $MT''N$, shown in the vertical projection (Fig. 2), would be obtained. The area, $MT''N$ (Fig. 2) evidently equals the original loaded area. To take advantage of this "transformed" area the following graphical construction is proposed: The stressed condition at Point O would not change if the horizontal loaded area, A, weighing p units per square unit of area, is replaced by the transformed area; also if A of the same unit weight, p, is concentrated as a vertical line load between Points M and N. Actually, a hypothetical load acting at a point of an arc (Fig. 2), can be moved to any other point of the same arc without influencing, in any way, the value of the vertical pressure at Point O. This is because the values of ρ and α to be used in Equations (1) and (4) for computing the vertical pressure, are the same at all points of that arc. Hence, for computing the vertical pressure at Point O, all the loads acting on an arc (Fig. 2) may be concentrated at an arbitrary point along that arc. This is precisely what is done in constructing the "transformed area" by bringing all the loads of different arcs (Fig. 2) to their respective points of intersection with an arbitrary line, $M'N'$. An elementary strip, dA, of that vertical figure, produces an elementary vertical pressure, dp_z, at Point O, which can be determined using Equations (1) and (4):

$$dp_z = \frac{n\,[p\,dA]}{2\,\pi\,\rho^2}\cos^n\alpha \quad\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

or, taking into consideration the fact that $\rho = \overline{OT'} = \frac{z}{\cos\alpha}$:

$$dp_z = \frac{n\,p\,dA}{2\,\pi\,z^2}\cos^{n+2}\alpha \quad\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The vertical pressure, p_z, at Point O, would be obtained from Equation (8) by summation, thus:

$$p_z = p\,\frac{n}{2\,\pi}\,\frac{1}{z^2}\,\Sigma\,dA\,\cos^{n+2}\alpha \quad\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

The area, $\Sigma\,dA\,\cos^{n+2}\alpha$, which will be termed the "reduced" area, A_r, may be determined as follows: An arbitrary vertical distance, O-O' is taken as a unit, and a graph of $\cos^{n+2}\alpha$ plotted. For doing so, radii vectors emanating from Point O and making different angles, α, with the vertical are drawn, and distances equal to $\cos^{n+2}\alpha$ are plotted along each radius vector. The ends of the radii vectors would then form a smooth curve which is the graph in question. For extensive use a table of values of

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[6] *Die Bautechnik*, Vol. 6, No. 17, p. 231 (1928).

$\cos^{n+2}\alpha$ will be found helpful. The following tabulation shows, for example, values of $\cos^{n+2}\alpha$ for the case in which the Boussinesq formula holds ($n = 3$):

α, in degrees	$\cos^5\alpha$	α, in degrees	$\cos^5\alpha$
10	0.927	50	0.110
20	0.735	60	0.031
30	0.487	70	0.005
40	0.264		

In Fig. 2 the unit vertical distance referred to, accidentally equals the depth, z. The point of intersection, X, of a radius vector, OT', is joined with the top of the graph, $\cos^{n+2}\alpha$ (Line $O'X$). An ordinate, such as $T'T''$, is then reduced, tracing $T''x$ parallel to $O'X$ to intersect the continuation of the radius vector, OT', at Point x. Plotting $T'y = T'x$ and repeating this operation for all points of the line, MN, the "reduced area," A_r, is obtained and measured by a planimeter. This value is then introduced into Equation (9) to replace $\Sigma\, dA \cos^{n+2}\alpha$:

$$p_z = p \frac{n}{2\pi} \frac{A_r}{z^2} \qquad \text{(10)}$$

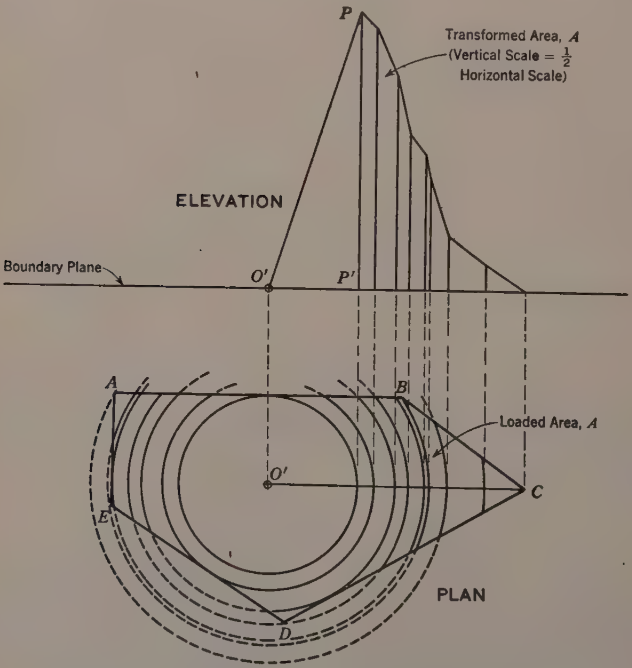
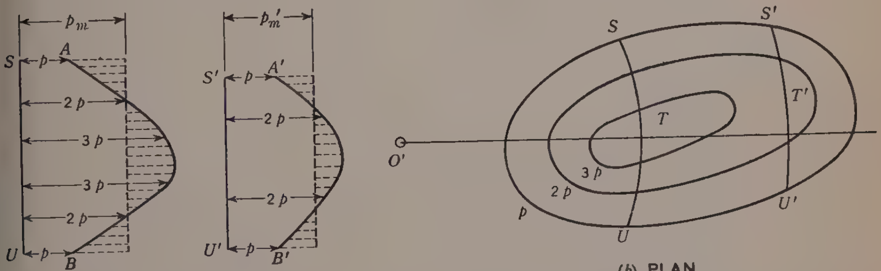


FIG. 3.—CONSTRUCTION OF THE TRANSFORMED AREA.

Since the ratio, $\frac{A_r}{z^2}$, is an abstract number, the scale of the drawing (see Fig. 2) has no bearing on the result. The value, $\frac{n}{2\pi}$, may be taken, from Table 1. If there is infinite pressure at a part of the loaded area (for instance, at its perimeter), the "reduced area" would also have infinite ordinates. In such a case it is difficult to measure the "reduced area" graphically, and the proposed method does not work.

In a particular case when the horizontal projection, O', of Point O, falls within the loaded area (Area $A\ B\ C\ D\ E$, Fig. 3), the largest possible complete circle is drawn within the area, from Point O' as a center. The ordinate, $P\,P'$, of the transformed area equals the length of the circumference of this circle; and $O'P$ is a straight line. The remainder of the procedure is as explained previously herein. As shown in Fig. 3, the construction may be confined to one side of Point O', plotting the ordinates of the "transformed" area as total lengths of the arcs of equal diameters (shown in solid lines in the plan, Fig. 3). Ordinates of the transformed area may be plotted on a reduced scale. In such a case the resultant vertical pressure, p_z, should be increased accordingly.

If the structure is non-uniformly loaded, for instance, as shown with isolines in Fig. 4(b) the problem may be easily reduced to the case of uniform loading with a given unit load, p. For doing so, first all arcs of the loaded area, such as $S\ T\ U$, are rectified, and pressures at each point plotted as ordinates. This procedure, as shown in Fig. 4(a) is analogous to the construction of a profile from a set of contour lines as in surveying. Let p_m be the average pressure acting at the arc, $S\ T\ U$; and if the latter is imagined in the form of a circular ring one unit wide, the total load carried by the arc in question would be $S\ T\ U \times p_m \times 1$. Since the vertical pressure p_z, at Point O caused by the arc, $S\ T\ U$, does not depend on the way in which pressures are distributed along that arc, but solely on the value of the total load carried, an arc of unknown length, x, loaded uniformly with an intensity, p, would cause the same pressure, p_z, at Point O, in which case the following condition is fulfilled: $S\ T\ U \times p_m \times 1 = x \times p \times 1$, from which $x = S\ T\ U\,\frac{p_m}{p}$. This means that all ordinates of the transformed area, are to be modified (increased in the case of Fig. 4) in the proportion, $\frac{p_m}{p}$, the value of p_m being individual for each ordinate.



(a) CROSS SECTIONS

(b) PLAN

FIG. 4.—PRELIMINARY GRAPHIC OPERATIONS IN THE CASE OF NON-UNIFORM LOADING.

Two-Dimensional Problem

In Fig. 5 a two-dimensional problem is solved for the general case of a non-uniformly distributed load, all the pressures being finite. At each point of the loaded area, $M N$, there is a vertical unit load, p_1, and the values of the ratio, $\frac{p_1}{p}$, are plotted as ordinates, such as $T' T''$. In this manner an area, $M T'' N = A$, is obtained which is analogous to the "transformed" area in the three-dimensional problem. The total load acting along $M N$, equals $p A$; and the elementary vertical, $p\ dA$, applied at Point T', produces, according to Equation (5), an elementary vertical pressure, dp_z, at Point O, equal to:

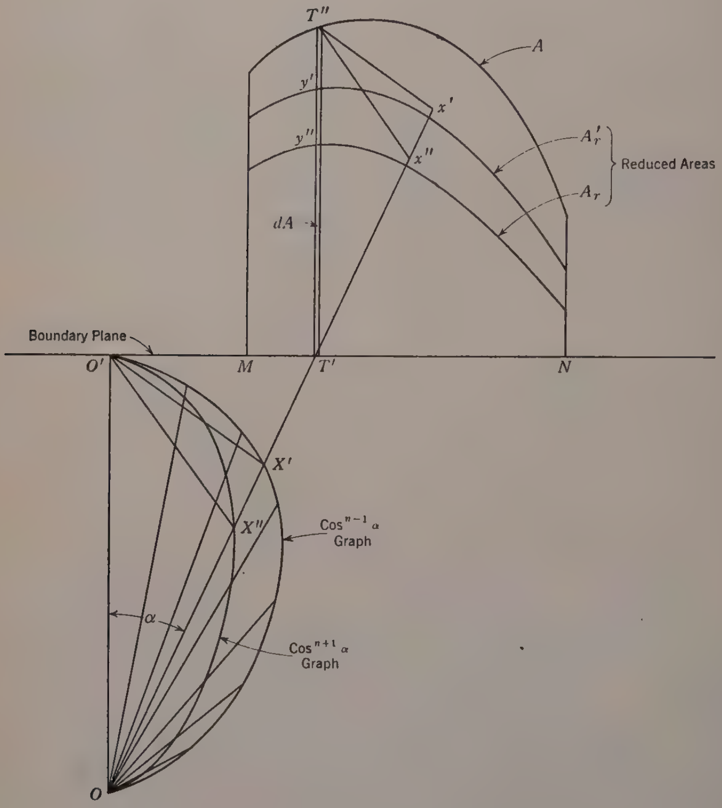


Fig. 5.—Vertical Pressure at a Point Determined by the Reduced Area Method (the Two-Dimensional Problem).

$$dp_z = \frac{n_1\,[p\,dA]}{\pi\,\rho}\cos^n\alpha = p\,\frac{n_1}{\pi}\,\frac{1}{z}\,[dA\,\cos^{n+1}\alpha]\;\ldots\ldots\ldots\ldots(11)$$

Equation (11) is derived directly from Equation (5). Following a procedure analogous to that used when discussing the three-dimensional problem:

$$p_z = p \, \frac{n_1}{\pi} \, \frac{A_r}{z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which A_r is the "reduced" area. To construct the reduced area, A_r, it is necessary to use the graph of $\cos^{n+1} \alpha$. For instance, if the Boussinesq formula holds, the equation would be $\cos^4 \alpha$. The values of $\frac{n_1}{\pi}$ may be taken from Table 1. If Point O is located somewhere under the loaded strip, $M\,N$, a vertical line passing through O (Fig. 5) would divide the drawing in two parts. A graph, $\cos^{n+1} \alpha$, is constructed separately for each part, the vertical line passing through O being the axis of symmetry of this double graph. The remainder of the procedure is as explained previously.

To determine the horizontal pressure, p_h, at Point O (Fig. 5), the sum of the principal stresses, s, should be found first, and then the vertical pressure p_z, subtracted from it. Substituting $p \, dA$ for p in Equation (6) and remembering that $\rho = \frac{z}{\cos \alpha}$, the elementary stress at Point O would be:

$$ds = p \, \frac{n_1}{\pi} \, \frac{1}{z} \, dA \, \cos^{n-1} \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Hence, the sum of the principal stresses, obtained from Equation (13) by summation, is:

$$s = p \, \frac{n_1}{\pi} \, \frac{A'_r}{z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which A'_r is the "reduced" area, constructed by using the graph, $\cos^{n+1} \alpha$. If the Boussinesq formula holds, the graph of $\cos^2 \alpha$ is to be used. Finally:

$$p_h = s - p_z = p \, \frac{n_1}{\pi} \, \frac{(A'_r - A_r)}{z} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

Since both ratios, $\frac{A'_r}{z}$ and $\frac{A_r}{z}$, are abstract numbers, the scale of the drawing (Fig. 5) has no bearing on the results. To make this fact clearer, it is necessary to recall that Area A is the sum of the products of dimensionless ratios, $\frac{p_1}{p}$, multiplied by certain widths; hence, A is a length, dimensionally.

The problem of finding the horizontal pressure, p_h, in the case of a three-dimensional stress distribution is more complicated than that solved in Fig. 5 for the case of a plane stress distribution and will not be discussed herein.

Normal stresses under non-rigid structures can also be determined by using the properties of the "angle of visibility" of the foundation. Application of this method to the study of plane stress distribution within a elastic isotropic mass has been discussed by the writer elsewhere.[7]

Example.—Fig. 6 illustrates the determination of the vertical pressure p_z, at a depth z = 100 ft, under the entering angle, $\overline{X}$, of a structure uni

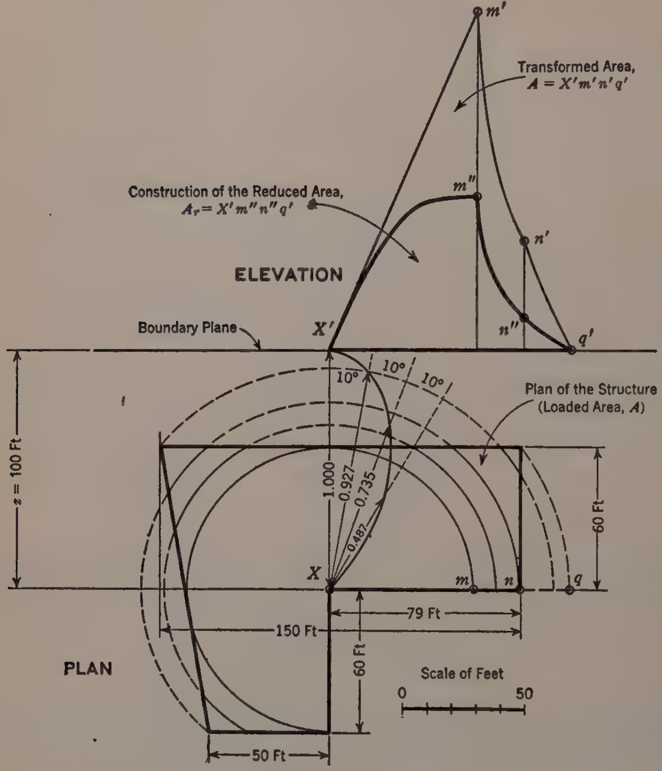


FIG. 6.—APPLICATION OF THE REDUCED AREA METHOD.

formly loaded with 2.5 tons per sq ft. The value of the reduced area A_r constructed under the assumption that the Boussinesq formula holds, is 6 56 sq ft, and the vertical pressure: $p_z = p \frac{n}{2\pi} \frac{A_r}{z^2} = 2.5 \times 0.477 \times \frac{6\,560}{100 \times 10}$ = 0.78 ton per sq ft. Note that the vertical scale used to construct both the transformed and reduced areas in Fig. 6 is one-half the horizontal.

PART II.—PRESSURES UNDER RIGID FOUNDATIONS

In the case of an actual structure, soil reactions are in equilibrium, no with unit loads acting at the loaded surface as assumed in Part I, but with stresses within the structure close to the surface of contact with the earth It follows that for finding stress intensities within the earth mass beneath a foundation, it is first necessary to establish the manner in which stresses are distributed at the base of the given structure and to consider the surface of contact of the structure with the earth as being loaded with those stresses acting as unit loads. The method which has been discussed in Part I, will be then fully applicable. Unfortunately, the problem of stress distribution at the base of a structure has not yet been solved definitely. Therefore, in presenting Part II of this paper, the writer limits his task to the review of available information in this field, both theoretical and experimental, and advances several ideas of his own. This discussion will be limited to uniformly loaded foundations only.

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[7] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1288.

RIGIDITY OF STRUCTURES AND EARTH MASSES

A structure will be termed "absolutely rigid" if it can be visualized as consisting of thin vertical slices so strongly bound together that they act as a unit. An opposite case would be a "non-rigid structure," which can be visualized as consisting of thin vertical slices with no lateral bonds so that each of them can settle down individually. The degree of rigidity in actual engineering structures is some value between these two limits. To form an accurate idea of the rigidity of the entire structure one should know how to express this rigidity numerically. At the present time this cannot be done in all cases.

Earth masses may also possess varying degrees of rigidity which could be expressed by a number as, for instance, the coefficient of pressure at rest, K, or the concentration factor, n. A mass that develops considerable horizontal stress under vertical load at rest (such as clays) may be considered as non-rigid. Compact sands, on the contrary, may be considered as rigid masses. If one knew how to express the rigidity of both the structure and the earth mass by comparable coefficients, one could attempt to attack the problem of the mutual rigidity of the two masses in contact, replacing the structure with an "equivalent mass." Because of the uncertainty of the manner of expressing the rigidity, however, both structures and earth masses will be identified in this paper, only as "rigid" and "non-rigid," the former approaching the state of absolute rigidity, as already defined.

If the earth mass supporting a structure is more or less homogeneous and isotropic to a considerable depth, the settlement of the structure is due to strains which gradually decrease with the depth. This is a case of "contact settlement" when the seat of settlement resides close to the base of the structure. If there is a soft layer at a certain depth, the structure may undergo an additional settlement owing to consolidation of that soft layer. It may be said that in this case the seat of settlement resides "in deeper strata."

When a deformable body moves through a medium offering resistance, its action is definable in two steps: (*a*) It moves as a non-deformable solid body; and (*b*) resistances of the medium produce deformations (or strains) in it. In a similar manner, a contact settlement is characterized by: (I) A downward movement of the structure; and (II) deformations of the structure due to interaction with the earth mass (Fig. 7). To understand the nature of these deformations the structure will first be assumed non-rigid; thus the phenomena at the surface of contact would occur practically as if the structure were replaced by a non-rigid earth mass. Afterward, rigidity will be restored to the structure, and corresponding changes in the stress distribution over the base of the foundation will be discussed. In a general way, stresses close to the surface of contact of two masses are the same in both masses, according to the well-known physical law of equality of action and reaction. Strains, however, are different because the less resistant of the two masses should yield. Hence, according to these considerations, a rigid mass under a non-rigid structure should bend the latter upward, as shown diagrammatically in Fig. 7(*c*). A case of settlement of this type was reported by Dr. Scheidig[8] when some strucures constructed on loose sand, settled at the edges. The writer understands that the buildings in question were erected rather hastily. Settlement I (Fig. 7(*c*)) occurred at once, owing to the compression of the loose sand; and probably when the latter became compact, the insufficiently rigid masonry was deflected upward. It should be noticed that if a loaded thin elastically isotropic layer is underlaid with rock or similar material, the formation of cupolas is also possible.[9] The writer understands, however,

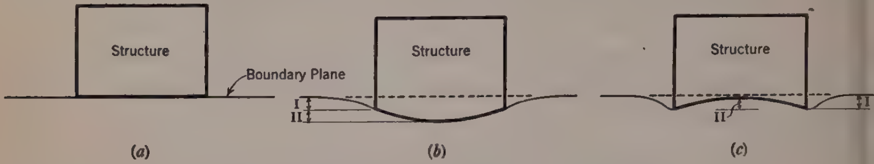

FIG. 7.—EXAMPLES OF CONTACT SETTLEMENT.

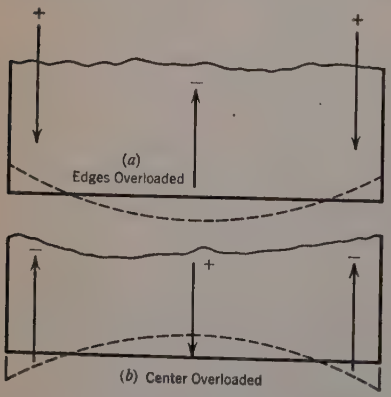

FIG. 8.—ACTION OF ABSOLUTELY RIGID STRUCTURE.

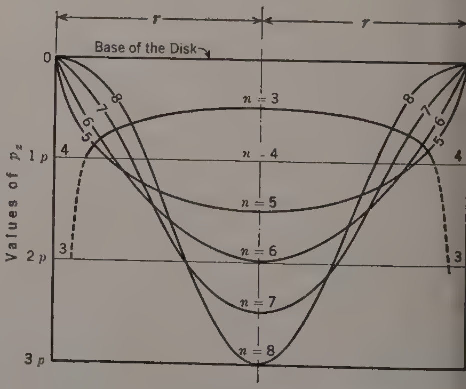

FIG. 9.—GRAPHICAL REPRESENTATION OF EQUATION (17).

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[8] "Die Berechnungsgrundlagen durchgehender Fundamente und die neuere Baugrundforschung", *Die Bautechnik*, Vol. 9, No. 19 (1931).

[9] "Theorie der Setzungen von Tonschichten", von Charles Terzaghi, M. Am. Soc. C. E., and O. K. Frőhlich, p. 8 (1936). Other references on the subject are also given in the footnote, pp. 8 and 9 of that book.

that there was no such underlying stratum in the case of Dr. Scheidig's observations, and that the edge settlement in this case should be explained by the inter-penetration of the masses in contact.

Although the problem shown in Fig. 7(*c*) cannot be solved using methods of the theory of elasticity, the latter furnishes all necessary data for the problem of settlement of a non-rigid structure of finite size, supported by an equally non-rigid mass (Fig. 7(*b*)). With Fig. 7(*b*) and Fig. 7(*c*) in mind it is now easy to determine the character of stress distribution at the base of a rigid structure, at least, qualitatively. In order to keep the base of the structure even, additional forces should be applied. Thus, an absolutely rigid structure may be over-loaded at the edges (Fig. 8(*a*)), or at the center (Fig. 8 (*b*)), according to the rigidity of the underlying mass (a non-rigid underlying mass in the former case and a rigid mass in the latter). Fig. 8 (*b*) furnishes an explanation as to why, in all loading experiments on sand masses, the maximum stress in the soil is shown under the center of the loading plate.

DISTRIBUTION OF NORMAL STRESSES UNDER AN ABSOLUTELY RIGID FOUNDATION

The problem of an absolutely rigid structure built on elastic isotropic soil has been solved by methods involving the theory of elasticity. Boussinesq[10] and later Schleicher[11] determined the stress distribution at the base of a rigid round disk placed on an elastic isotropic mass from displacements of its points. Boussinesq's basic assumptions are: (1) That there is no friction at the plane of contact; and (2) the plane of contact is horizontal. Professor Michael A. Sadowsky,[12] of the University of Minnesota, applied the potential theory to solve the three-dimensional problem. According to these investigators, if r is the radius of the disk in question, the pressure at a point distant, $c\,r$, from the center of the disk, would be:

$$p_z = p\,\frac{1}{2\sqrt{1-c^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The pressure, p_z, equals $0.5\,p$ at the center of the disk ($c = 0$) and becomes infinite at the edges ($c = 1$). Since there are no absolutely rigid structures, even the slightest settlement at the edge of the loaded disk relieves the strain considerably, so that practically all stresses at the base of the loaded disk are finite. The influence of rigidity of a circular foundation slab in the distribution of pressures on the contact surface, has been considered by H. Borowicka[13] who assumed that both the structure and the earth mass are elastically isotropic with different values of the modulus of elasticity and Poisson's ratio. To express the stress distribution in a case of an absolutely rigid round disk placed at the top of an earth mass, with a concentration factor, n, the writer proposes the following empirical formula (see Fig. 9):

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[10] "Application des Potentials a l'Etude de l'Equilibre et du Movement des Solides Elastiques", Paris, 1885, p. 158; also, "Theory of Elasticity", by S. Timoshenko, p. 338.

[11] "Die Verteilung der Bodenpressungen unter starren Gründungskörpern", *Der Bauingenieur*, Vol. 14, Nos. 17–18, p. 242 *et seq.* (1933); also. *Zeitschrift für angewandte Mechanik*, Vol. 8 (1928).

[12] "Distribution of Pressure Concentrated on a Small Surface", *Transactions*, Am. Soc. M. E., Vol. 54, Paper APM 54–21, p. 221 (1934).

[13] *Proceedings*, International Conference for Soil Mechanics and Foundation Engineering, Vol. II, Paper E-8.

$$p_z = p\left(\frac{n}{2} - 1\right)\left(1 - c^2\right)^{\frac{n}{2}-2} \qquad (17)$$

Substituting $n = 3$ in Equation (17), the resulting form is identical to Equation (16). The value, $n = 6$, results in a parabolic distribution very similar to that found in numerous loading experiments on sand masses as reviewed elsewhere:[14]

$$p_z = 2\,p\,(1 - c^2) \qquad (18)$$

Should the concentration factor, n, of the constrained earth mass increase to 8, the stress distribution at the base of the experimental disk would be defined by the curve:

$$p_z = 3\,p\,(1 - c^2)^2 \qquad (19)$$

and this creates a pressure at the center of the disk equal to 300% of the average pressure, p, which is quite close to the result obtained by M. L. Enger,[15] M. Am. Soc. C. E.

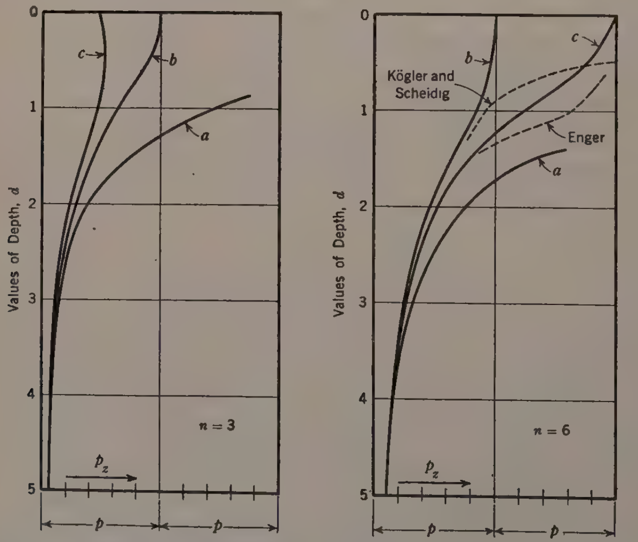

FIG. 10.—VERTICAL PRESSURES AT VARIOUS DEPTHS ALONG THE CENTER LINE OF A LOADED CIRCULAR DISK

Equation (17) has been designed on the assumption that the lower value of the concentration factor, n, equals 3. This is true of the isotropic masses for which the basic formulas given in the "Introduction," have been designed. Other geological conditions (for example, that of a hard layer underlaid by a soft one) are not covered by the formulas of this paper, and the solution of the corresponding problems should be sought elsewhere.

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[14] "Distribution of Stresses Under a Foundation", by A. E. Cummings, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1077.

[15] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 372.

The problem of stress distribution under a rigid structure is complicated by the formation of the so-called "disturbed zone." It follows from the small-scale experiments that the cross-section of this zone under a circular disk is semi-circular or ellipsoidal with a top segment removed. These results cannot yet be generalized for the case of full-sized structures, however, because of the lack of corresponding observations. In order to investigate to what depth the rigidity of a loaded plate in a small-scale experiment affects the earth mass, Fig. 10 has been prepared. For two soils, with concentration factors $n = 3$ and $n = 6$, respectively, curves representing the vertical pressure, p_z, at various depths along the center line of a round loaded disk, have been traced. Pressures p_z have been plotted horizontally. The load is the same in all cases, but distributed differently: Curves (*a*) correspond to a concentrated load acting directly at the surface of the soil; Curves (*b*) correspond to uniform distribution; and Curves (*c*) correspond to the case of an absolutely rigid plate, respectively. For the latter case, Equations (16) and (18) have been used (see Appendix II for mathematical details). At a depth of about twice the diameter of the loaded part ($z = 4r$ in Fig. 10), Curves (*a*), (*b*), and (*c*) practically coincide, which accounts for the well known statement that, at that depth, a loaded plate acts practically as a concentrated load. The average experimental curves of Kögler and Scheidig and Enger[16] (see Fig. 10), are located partly to the right of Curve (*c*); and this means that in the upper part of the experimental mass, approximately down to a depth, r, the concentration factor, n, exceeds 6 for which value the curve, *c*, in Fig. 10 has been traced. Afterward, the experimental curves swing rapidly to join Curve (*b*); and at a depth of about $2r$ there is no longer any influence of the rigidity of the loaded plate—at that depth the mass behaves as if the base of the experimental disk were uniformly loaded (Curve (*b*), Fig. 10).

Undoubtedly, the stress distribution at the base of an infinitely wide rigid structure should be uniform; it cannot be otherwise. To reconcile this simple logical statement with the results previously obtained, Fig. 11 has been prepared. The stress distribution at the base of a rigid narrow structure is represented in Fig. 11(*a*) and Fig. 11(*c*), for the cases of a non-rigid and a rigid earth mass, respectively. The stress distribution under a wide rigid structure is tentatively shown in Fig. 11(*b*) and Fig. 11(*d*), for the cases of a non-rigid mass and a rigid earth mass, respectively. It should be noticed that the curve in Fig. 11(*b*) is very close to that found theoretically by K. Wieghart[17] for the case of a beam supported by an elastically isotropic mass.

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[16] The average values in question are those computed in the paper entitled "Distribution of Stresses Under a Foundation", by A. E. Cummings, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1078.

[17] Handbuch der Physik, Edited by R. Grammel, Vol. 6, pp. 180–181 (1928); for original work by K. Wieghart, see *Zeitschrift f. angew. Mathematik und Mechanik*, Vol. 2 (1922).

PRESSURES UNDER ACTUAL ENGINEERING STRUCTURES

To check theoretical considerations, pressure at the base of actual structures should be measured. This can be done by embedding pressure-measuring cells at the base of the structure during its construction. There are only a few published examples of such measurement, although such embedments have been reported from many places. From the three cases known to the writer one refers particularly to sewers[18] and two others will be discussed subsequently.

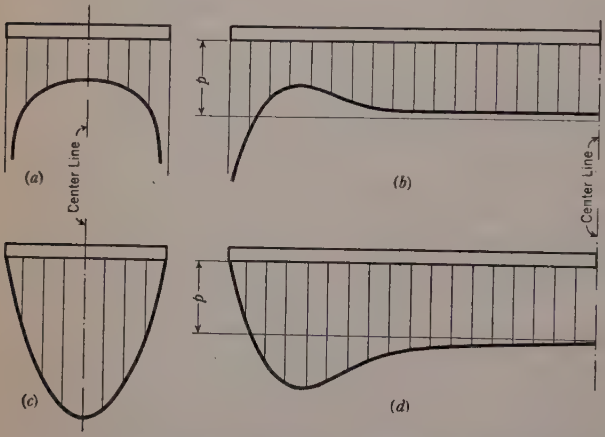

FIG. 11.—STRESS DISTRIBUTION AT BASE OF RIGID FOOTINGS, BOTH NARROW AND WIDE (A TENTATIVE SKETCH).

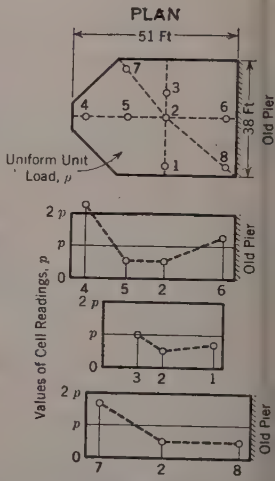

FIG. 12.—PRESSURES BENEATH A BRIDGE OVER THE RHINE RIVER IN GERMANY.

During the construction of the new bridge over the Rhine River, connecting Mannheim and Ludwigshafen, Germany, eight Goldbeck cells were placed beneath a pier,[11] and it was discovered that the foundation was overloaded at the edges (see Fig. 12). The foundation material was gravelly, and since European engineering thought had been accustomed to the idea that in the case of similar foundations the maximum pressure should be at the middle, there was an interesting discussion of the case.[19] The uncertainty was emphasized by the fact that the new pier was constructed close to the old one, which could have a certain influence on the stress distribution. In the case under consideration, the pier was founded on a caisson. Soil reaction could not lift the heavily loaded edges of the caisson, and, apparently, the foundation material was forced somewhat, into the concrete fill of the chamber. Consequently, the stress at the middle of the structure was relieved (see Fig. 7(*c*)). The writer believes that a similar condition may occur in other cases of caisson foundations if the latter may be considered as non-homogeneous structures consisting of a rigid shell containing a less resistant mass.

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[18] "Observed Soil Pressures on the Deep Sewers of the North Toronto System", by C. R. Young, M. Am. Soc. C. E., and W. B. Dunbar, *Bulletin No. 145*, Univ. of Toronto (1935).

[19] "Sohldruckverteilung unter Pfeilern", by F. Kögler, *Der Bauingenieur*, Vol. 14, Nos. 37–38, p. 473 *et seq* (1933); also discussions by Messrs. Scheidig and Schleicher, pp. 482 and 483.

About 1933 a building was constructed by the Agricultural and Mechanical College of Texas.[20] Six Goldbeck cells were embedded under a footing as shown in Fig. 13. The soil beneath the building is clay with small varying quantities of sand and coarse material. The maximum pressure during the construction was at the center of the footing (Cell No. 4), but this shifted gradually during a period of about seven months to Cell No. 3. It might be concluded, at first, that the concentration of stresses near the middle of this structure, as shown by Fig. 13, is in disagreement with the hypothesis advanced, as, for instance, with the stress distribution shown in Fig. 11(*a*). Apparently, Fig. 11(*a*) predicts an overloaded edge under the given soil conditions. The confusion is clarified, however, if one realizes that the footing action is not included in the scope of this paper, which discusses only cases of the interaction between two masses. In the case under consideration, however, there is an interaction between three masses: A column, a base, and an earth mass (see

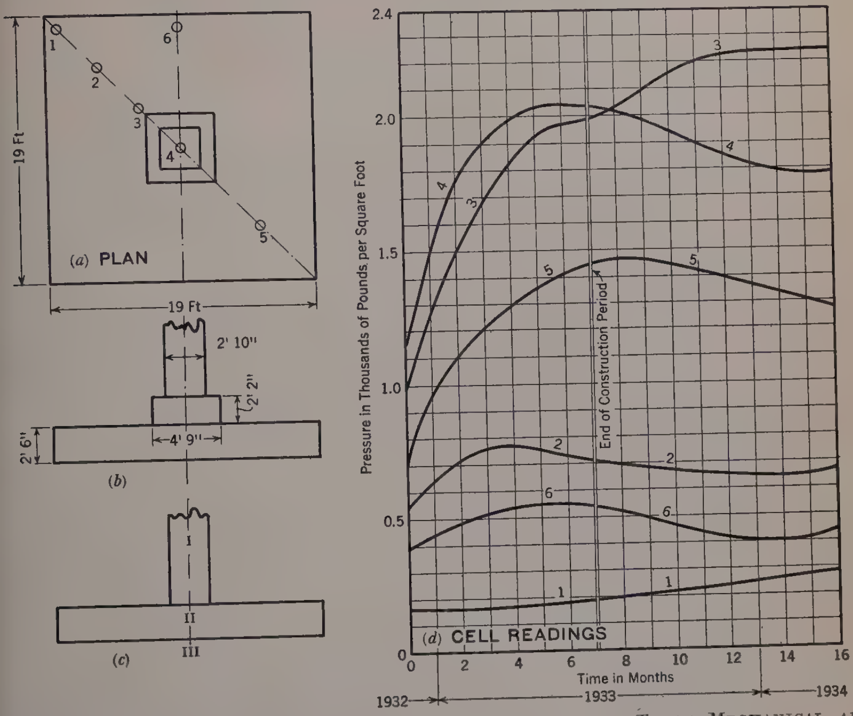

FIG. 13.—PRESSURES BENEATH A FOOTING OF A BUILDING AT TEXAS MECHANICAL AND AGRICULTURAL COLLEGE.

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[20] "The Distribution of Soil Pressure Beneath a Footing" by F. E. Ciesecke M. Am. Soc. C. E., and others, *Bulletin No. 43*, Texas Eng. Experiment Station, 1933.

Fig. 13(*c*)). Should the rigid mass, I, Fig. 13(*c*), act directly at the surface of the mass, III, the edges probably would become overloaded; but the inclusion of the intermediate mass, II, apparently changes the situation A very important conclusion from the Texas experiment is the possibility of pressure fluctuation at the base of a structure, which is probably due to a progressive change in the physical properties of the earth mass.

OUTLINE OF THE NECESSARY RESEARCH WORK

The most important necessary research in this field may be stated in four items:

(1) Since hypotheses and speculations concerning the interaction between the structure and the earth mass are practically worthless unless checked against field observations, it is urgent to increase the number of such observations. Pressure-measuring cells should be embedded under new structures in all cases where interesting results concerning such interaction may be expected.

(2) A classification of engineering structures concerning the influence of their rigidity upon their interaction with the earth mass, should be elaborated. Attempts should be made to express the rigidity of the structure and the underlying earth mass with comparable numerical coefficients. Investigators should determine the cases in which structures act on the underlying ground as an "equivalent earth mass" and when this action is of a more complicated nature.

(3) Further study of the theory of the "concentration factor," n, is of importance. Attempts should be made to develop methods of measuring the value of the concentration factor in the field before (and not after) a structure is built.

(4) Considerable development of stress distribution theories was noted during the period, 1934 to 1936. Unfortunately, these theories refer only to the so-called "semi-infinite" masses, such as those shown in Fig. 1. It is necessary to extend these theories in the case of "finite" masses, such, for example, as embankments. Some progress in this field has been made since the introduction of photo-elastic analysis. Data analogous to those obtained in the study of the properties of elastic bodies in contact should be obtained for "finite masses" of all kinds.

Most of the research outlined in Items (1) to (4) cannot be done by soil mechanics investigators alone. It requires, also, the most intensive cooperation of structural engineers.

SOME REMARKS ON THE DESIGN OF FOUNDATIONS

Bending Moment.—When a structure is designed under the assumption of uniformly distributed soil reaction (as generally done in practice) there is no bending in the loading plate except in cantilever units; but if the "picture," afforded by Fig. 11(*b*) and Fig. 11(*d*), is correct, only the central part of a wide structure can be designed under that approximate assumption. According to Fig. 11(*b*) and Fig. 11(*d*) there should be bending moment, positive or negative, near the perimeter of a bearing surface, the sign of the bending moment depending on the physical properties of the local soil.

Settlement of a Structure Due to Consolidation in Deeper Strata.—To design a structure so that it will not fail by settlement, the designer should be aware of the geological condition of the earth mass to a considerable depth, say, about twice the width of the structure. This is because a possible consolidation of a deep soft layer may cause a serious settlement. Consolidation in deeper strata, however, has no effect on the interaction of the structure and the earth mass. In this case the entire situation is controlled by the same physical law which permits, for instance, the playing of games of ball or cricket on a moving steamship without disturbance. In an analogous sense, when consolidation in deeper strata begins, the entire mass, including the structure, would settle, and there is no reason whatever to believe that the stressed condition at the base of the structure would be disturbed. Settlement in such a case is assumed to be uniform. A similar idea, however, in a somewhat different form, has been advanced by Professor A. Casagrande.[21]

Interrelation of Stresses.—Serious thought should be given to the question of whether the actual method of designing a structure without taking into consideration the influence of the foundation on the stress distribution in the structure itself should be discontinued. Undoubtedly, the structure and the earth mass underlying it form a single statically indeterminate system; and the stress distribution in the mass influences that in the structure, at least at its lower part. This principle has been realized by dam builders; and it is to be hoped that an analogous procedure will be adopted for other branches of structural engineering.

CONCLUSIONS

1.—Stresses within a semi-infinite isotropic mass (see Fig. 1), caused by a concentrated force acting at the boundary, can be determined by using Equation (1). In the case of a two-dimensional (plane) stress distribution, Equation (6) should be used.

2.—Stresses caused by loads distributed at the boundary of an isotropic mass may be determined by superposition.

3.—The "reduced area method" supplies a means of determining pressures in an isotropic mass beneath uniformly loaded foundations. It consists in subdividing the loaded area into circular zones, rectifying them, to obtain the "transformed area" and reducing the latter graphically, according to the position of the load with respect to the point where pressures are to be determined.

4.—The "reduced area method" can also be applied in the case of non-uniformly loaded isotropic masses, if all unit loads are finite. Otherwise, the graphical measurement of the "reduced area" is difficult.

5.—The degree of "contact settlement" that may be expected of a structure built at the top of an earth mass is chiefly controlled by the mutual rigidity of both masses in contact and perhaps by other factors, such as the size and the shape of the structure.

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[21] *Journal*, Boston Soc. of Civ. Engrs., Vol. 23 (1936), p. 15.

6.—The phenomenon of "contact settlement" of a non-rigid structure supported by a rigid earth mass consists in a downward movement, and in an upward deflection, of the structure.

7.—In the case of a narrow, absolutely rigid, structure, there is: (*a*) An overloading at the edges in the case of a non-rigid (for instance, a clay) foundation; or (*b*) an over-loading at the center in the case of a rigid (for instance, a sand) foundation.

8.—An empirical formula, Equation (17), concerning the stress distribution at the base of a narrow rigid structure, has been proposed.

9.—In the case of a wide engineering structure, there should be a certain tendency toward a uniform stress distribution at the center of the foundation. Apparently, the edges of a rigid structure are always overloaded, and there is always bending along its perimeter. Such moment will be positive or negative, depending on the physical properties of the soil beneath the foundation.

10.—There is a "disturbed zone" under a rigid structure apparently possessing a higher concentration factor than the remainder of the mass. The state of constraint along the center line of a round loaded plate in small-scale experiments goes only to a depth equal to one-half the width of the loaded area. Between this depth and that equal to twice the width of the loaded area, the stresses in the mass act as if the load were uniformly distributed through the foundation. At, and beyond, a depth twice the width of the loaded area the loading plate acts as a concentrated load.

11.—The stressed condition at the surface of contact of the structure with the earth mass does not depend on the consolidation in deeper strata.

12.—Field observations available concerning the stress distribution at the base of a structure, are insufficient. Hence, statements advanced in Part II of this paper (except those based on firm physical laws) are rather of a hypothetical nature. Further research is needed before they can be generalized and adopted unconditionally.

APPENDIX I

DETERMINATION OF THE VALUE OF THE "PLANE CONCENTRATION FACTOR," n_1

The given force, *P*, and another unknown force, *X*, acting along the direction of the radius vector, *A O*, in Fig. 14, would produce the same stress at Point *O*, if Equation (20) is satisfied:

$$\frac{n\,P}{2\,\pi\,\rho^2}\cos^{n-2}\alpha = \frac{n\,X}{2\,\pi\,\rho^2} \qquad (20)$$

from which,

$$X = P\cos^{n-2}\alpha \qquad (21)$$

Equation (21) holds also if a line load, $\bar{p}$, is substituted for the concentrated force, P. Hence, the value of the plane stress, s, at Point O, caused by the action of a line load, $\bar{p}$, at A (Fig. 14(a)) equals the vertical pressure at Point O' (Fig. 14(b)), distant vertically ρ from Point A, and subject to the action of a line load, $\bar{p}\cos^{n-2}\alpha$, at the boundary. In Fig. 14(b) the distance, $A\,M = \rho\tan\phi$, and $M\,M'$ is its differential. Hence, the value of this stress, s (see Equations (1) and (6)) is:

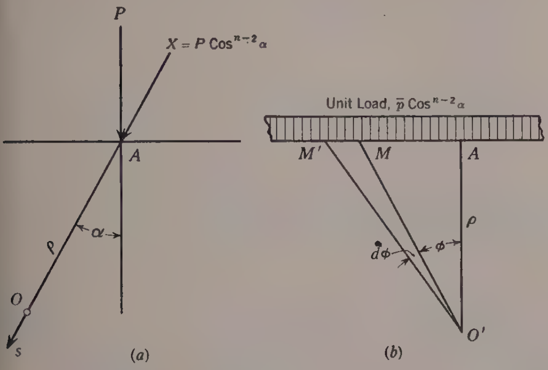

FIG. 14.

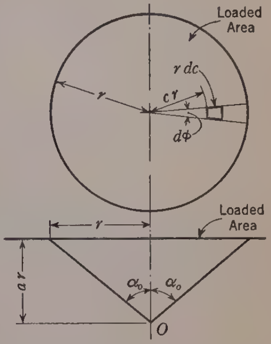

FIG. 15.

$$s = 2\int_0^{\frac{\pi}{2}} \frac{n\left[\bar{p}\cos^{n-2}\alpha\right]\left[\frac{\rho}{\cos^2\phi}\,d\phi\right]}{2\,\pi\left(\frac{\rho}{\cos\phi}\right)^2}\cos^n\phi = \frac{n_1\,\bar{p}}{\pi\,\rho}\cos^{n-2}\alpha$$

in which:

$$n_1 = n\int_0^{\frac{\pi}{2}}\cos^n\phi\,d\phi \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

In turn, the integral in Equation (22) equals:

$$\int_0^{\frac{\pi}{2}}\cos^n\phi\,d\phi = \frac{\sqrt{\pi}}{2}\,\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}+1\right)} \quad\ldots\ldots\ldots\ldots(23)$$

in which Γ is the gamma function. If the concentration factor, n, is not an integer, the value, n_1, is determined by using Equation (23). Otherwise, it is better to compute that value as follows: When n is an odd number, Equation (22) becomes:

$$n_1 = \frac{2\times4\times6\ldots(n-1)}{1\times3\times5\,..\,(n-2)} \quad\ldots\ldots\ldots\ldots\ldots\ldots(24a)$$

and, when n is an even number:

$$n_1 = \frac{1 \times 3 \times 5 \ldots\ldots (n-1)}{2 \times 4 \times 6 \ldots\ldots (n-2)} \, \frac{\pi}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24b)$$

Equations 24(a) and 24(b) check with formulas of plane stress distribution developed by Dr. O. K. Fröhlich.[22]

APPENDIX II

Formulas for Tracing Curves in Fig. 10

Equation (1) was used in plotting Curves (a), Fig. 10. Making $\alpha = 0$ and $\rho = a\, r$;

$$p_z = \frac{n\,[\pi\, r^2\, p]}{2\,\pi\,[a\, r]^2} = p\, \frac{n}{2}\, \frac{1}{a^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which a is a coeffcient $= \dfrac{\rho}{r}$.

The well-known formula[23] for the case of a uniformly loaded circular area was used in plotting Curves (b), Fig. 10; thus:

$$p_z = p\,(1 - \cos^n \alpha_0) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which α_0 is the angle formed with the vertical by the radii vectors joining the given point, O, with the perimeter of the loaded area (see Fig. 15).

Referring to Fig. 15 the area of the loaded element at the boundary is:

$$d\,A_c = [c\, r\, d\phi]\,[r\, dc] = r^2\, c\, dc\, d\phi \quad \ldots\ldots\ldots\ldots\ldots (27)$$

The vertical pressure in the case of constraint is to be determined: (a) Using Equation (16) in the case of $n = 3$; and (b), using Equation (18), when $n = 6$. Thus, if $n = 3$:

$$p_z = \int_0^{2\pi}\!\!\int_0^{1} \frac{\dfrac{3\,p}{2\,(1-c^2)^{\frac{1}{2}}}\,[r^2\, c\, dc\, d\phi]}{2\,\pi\,(a^2\, r^2 + c^2\, r^2)} \times \frac{a^3\, r^3}{(a^2\, r^2 + c^2\, r^2)^{\frac{3}{2}}}$$

$$= \frac{3}{2}\, p\, a^3 \int_0^1 \frac{c\, dc}{(1-c^2)^{\frac{1}{2}}\,(a^2+c^2)^{\frac{5}{2}}} \quad \ldots\ldots\ldots\ldots\ldots (28)$$

Substituting t^2 for $(1 - c^2)$ and remembering that the limits of t are 1 and 0, when $c = 0$ and $c = 1$, respectively:

$$p_z = -\frac{3}{2}\, p\, a^3 \int_0^1 \frac{dt}{(a^2 + 1 - t^2)^{\frac{5}{2}}} = p\, \frac{1 + 3\,a^2}{2\,(a^2+1)^2} \quad \ldots\ldots\ldots (29)$$

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[22] "Druckverteilung im Baugrunde", p. 28 (1934).

[23] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), Equation (23), p. 1090.

and if $n = 6$;

$$p_z = \int_0^{2\pi}\int_0^1 \frac{6\,[2\,p\,(1-c^2)]\,[r^2\,c\,dc\,d\phi]}{2\,\pi\,[a^2\,r^2+c^2\,r^2]} \times \frac{a^6\,r^6}{(a^2\,r^2+c^2\,r^2)^3}$$

$$= p\left\{2 - a^2 + \frac{a^6}{(a^2+1)^2}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

To obtain this result, a new variable has been substituted for c^2.

APPENDIX III

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[24] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = loaded area; $d\,A$ = a small element of a loaded area; A_r = reduced area as applied to vertical pressure; A'_r = reduced area as applied to horizontal pressure; A_c = area of a circle;

a = a coefficient $= \frac{\rho}{r}$;

c = a coefficient to express the distance from the center of a circular loaded plate, in terms of its radius;

K = coefficient of pressure at rest, or the ratio of principal stresses in a non-loaded semi-infinite earth mass;

n = a concentration factor; n_1 = a plane concentration factor, n_1 being a function of n;

P = a concentrated vertical force acting at the boundary of an earth mass (see, also $\bar{p}$);

p = pressure per unit area; a uniformly distributed load; p_m = average pressure acting along an arc of the loaded area; p_1 = non-uniformly distributed load; p_z = vertical unit pressure at any point; p_h = horizontal unit pressure at a point; $\bar{p}$ = a unit line load.

r = radius of a circular loaded plate, or disk; as a subscript, r, denotes "reduced";

s = unit stress, or sum of principal stresses;

t = a substitution factor $= (1 - c^2)$;

z = depth of a point below the ground surface;

α = angle of a force, or of a stress with the vertical; α_o = one-half the angle at the vertex of a regular cone;

Γ = gamma function;

ρ = radius vector of a point (distance from the origin);

ϕ = auxiliary angles in Figs. 14 and 15.

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[24] A. S. A.—Z10a—1932.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

WATER SUPPLY ENGINEERING[1]

PROGRESS REPORT OF COMMITTEE OF SANITARY ENGINEERING DIVISION

With the approval of the Executive Committee of the Division, this Committee has made its main business the reporting of engineering progress affecting water supply, including also at times notes of committee activities in the Society and other organizations. In the report[2] for 1935 notes were added listing some of the more important water supply projects which were under construction.

The Committee has had a feeling that its notes of progress have overlapped considerably from year to year, and this effect is increased at present by the inclusion in the program of the American Water Works Association at its Annual Convention, at Los Angeles, Calif., in June, 1936, of several very complete epitomes of progress in various branches of water supply engineering. It seems too soon to inflict another full report on the water supply group, whether sitting as members of the American Water Works Association, or as members of the Sanitary Engineering Division of the Society. The Committee, therefore, has made an attempt at brevity, although the inclusion of certain miscellaneous items of interest has defeated this original intention to a considerable extent.

WATER-WORKS CONSTRUCTION IN PROGRESS

Industrial Supply for Birmingham, Ala.—The City of Birmingham, Ala., has under construction as a project of the United States Works Progress Administration (W.P.A.) an industrial supply of 40 mgd from the Blackburn Fork of the Little Warrior River. The project involves a rock-fill, clay-faced dam, 200 ft high and 1 060 ft long, impounding 21 000 000 000 gal of water; a steel distributing pipe, 60 to 42 in. in diameter, aggregating about 122 000 ft in length; and 25 000 ft of 24-in. and 16-in. cast-iron pipe. Water rates are to be from 7 cts to 2.5 cts per thousand gallons.

Metropolitan Water District of Southern California, Supply from Colorado River.—Construction continues on this huge project. The Parker Dam across

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NOTE.—Written discussion of this report will be transmitted directly to the Chairman, for the information of the Committee.

[1] Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 21, 1937.

[2] *Proceedings*, Am. Soc. C. E., March, 1936, p. 355.

the Colorado River is under construction and the 392-mile aqueduct is more than one-half completed. This system will ultimately supply 1 000 mgd to Los Angeles, Pasadena, and such other counties and villages as may from time to time join the District. The City of San Diego is said to be studying the advisability of joining.

Orange Valley Supply for Los Angeles, Calif.—A tunnel, 11.3 miles long, is being driven 350 miles north of the City of Los Angeles to reinforce the flow of the Orange River Aqueduct (which was finished in 1913) with water from the Mono Basin. The impounding reservoirs in the Mono Valley include a reservoir in Long Valley, having a capacity of about 53 000 000 000 gal, and another reservoir at Grant Lake.

Moffat Tunnel at Denver, Colo.—The $10 000 000 Moffat Tunnel project has been completed to the point where a diversion to aid the Denver supply was made in 1936. During the year additional funds were made available by which soft waters from the west slope will be carried across to Denver in a 54-in. conduit under a 400-ft head, thus permitting the elimination of most of the pumping to high-service areas. The Denver work includes an earth dam 185 ft high, a 56-mgd filter plant, nearly 30 miles of 42-in. to 63-in. steel and reinforced concrete conduits, and an extensive system of canals, tunnels, and control works.

Milwaukee, Wis.—The 200-mgd filtration project at Milwaukee, Wis., is now about 80% under contract and will probably be completed in 1938. The costs are well within the original estimate of $4 800 000. All substructures, except the head-house, are complete. The filter substructure is about 75% complete, and most of the filter equipment is in place.

Cincinnati, Ohio, Plant.—Rehabilitation and enlargement of the plant at Cincinnati, Ohio, is in active construction. The old plant, designed and built in 1906 under the late George H. Benzenberg, Past-President, Am. Soc. C. E., is an outstanding example of engineering foresight, and the capability of a broadly designed plant to handle conditions that could not have been foreseen when the original plant was constructed.

Delaware River Supply for New York, N. Y.—On January 12, 1937, the City of New York opened bids for the sinking of thirteen shafts on an 85-mile tunnel. Seventeen more shafts will be required. The first step has thus been taken for adding to the present water supply of New York City, which has an estimated yield in a series of dry years of 1 075 mgd, a new supply of approximately 540 mgd. Of this quantity, 100 mgd is to come from the Rondout water-shed and the other 440 mgd from the East Branch of the Delaware River and the Neversink River which is also tributary to the Delaware. The present supplies of New York City will be equal to the estimated consumption in about 1939, and the new supply is estimated to take at least eight years to construct, which leaves New York City somewhat at the mercy of current rainfall for six years or more following 1939, unless some other temporary supply can be found in the meantime. A supply from Suffolk County is a possibility, although a change of sentiment in that County, and additional legislation, would be required before this supply could be made available.

The new supply from the Delaware River and from Rondout Creek involves the construction of Rondout and Neversink Reservoirs in the first stage, and of East Branch Reservoir in the second stage. The original studies provided for a taking of 660 mgd from the Delaware water-shed. This was cut down to 440 mgd by the decree of the United States Supreme Court in 1931 as a result of a suit which was participated in by the States of New York, Pennsylvania, and New Jersey. As a part of this litigation extensive surveys were made of the possible effect of the New York City withdrawals of water on the salinity of the Delaware River, as affecting water supplies taken from the river south of Philadelphia, Pa., and the shell-fish industry near the mouth of the river. New York City is required to deliver from its storage reservoirs certain quantities of water when the flow of the Delaware River is below a certain minimum, and the Supreme Court left for possible future determination whether this provision would be adequate to prevent increased salinity. New York City also has a right to ask the Supreme Court for additional water when required.

The proposed new aqueduct from Delaware and Rondout sources will connect with the West Branch Reservoir of the Croton System and terminate at Kenisco Reservoir, but a duplicate aqueduct, all in tunnel, will be constructed from Kenisco Reservoir southward to Hillview Reservoir, the large distributing reservoir. City Tunnel No. 1 starts at Hillview Reservoir passes through The Bronx, Manhattan, and over into Brooklyn. City Tunnel No. 2 recently put into service also starts at Hillview Reservoir but passes through The Bronx, across the East River into the Borough of Queens, and terminates in Brooklyn with a connection to Tunnel No. 1 at that place.

The New Springwells Pumping and Filter Plant at Detroit, Mich.—In 1932, the City of Detroit finished a complete and beautifully equipped pumping station and filter plant to furnish water for North and West Detroit and adjoining municipalities. Its capacity is about 300 mgd, and it receives its water through a 12-mile tunnel from 12 ft to 15½ ft in diameter. The low-lift plant has ten pumping units carrying a combined capacity of 435 mgd, six of these units having capacities of 60 mgd. The high-lift plant has an installed capacity of 568 mgd and will ultimately have a capacity of 680 mgd in sixteen units. Power is generated by a steam plant with turbo-electric generators. This plant has been out of service since its completion because consumption was greatly reduced by the depression.

Fort Smith, Ark.—A filtered supply has recently been completed in Fort Smith, Ark. The water is brought from Clear Lake Mountain Reservoir by a 23-mile welded-steel pipe line, 27 in. in diameter, and flows by gravity to a filter plant consisting of four 1.25 mgd units. The unique feature of this plant is a circular clear-water reservoir 88 ft in diameter and 22 ft deep, the outer wall of which consists of twelve segmental vertical arches convex inward spanning between twelve buttresses.

Colorado Springs, Colo., Steel-Faced Dam.—There is under construction at Colorado Springs, Colo., a steel-faced dam the supporting part of which consists of fill of disintegrated granite. A similar dam on Crystal Creek was completed in 1935. From these reservoirs, a pipe line with a maximum head of 1 909 ft will carry an average supply of 6 mgd from reservoirs on the slopes of Pike's Peak to Colorado Springs.

Improvements of Buffalo, N. Y., Water System.—Improvements are under way at Buffalo, N. Y., designed to unify the high- and low-service systems. Included in the program are three elevated tanks of 2 000 000-gal capacity, three electric-driven high-pressure fire pumps (3 000 gal per min), an addition to the clear-water reservoir at the filtration plant, of 10 500 ft of 36-in. trunk-feeder loop, and the reconstruction of many distribution mains.

Little Rock, Ark.—New Mountain Water Supply.—Well along toward completion for Little Rock, Ark., is a 14 000 000 000-gal reservoir, behind a rolled-earth dam, 115 ft high and 2 800 ft long, with a reinforced concrete pipe line 39 in in diameter, 32 miles long, including two reinforced tunnel sections, 1 100 ft and 1 500 ft, respectively, in length The pressure on the pipe line will vary from practically zero to 300 ft, with a provision for an extra head of 50 ft to permit possible future use of a booster pump. The new supply is on the Alum Fork of the Saline River. The average hardness of the new supply will be about 10 ppm as compared with several hundred parts of hardness and much iron, and, in the drought year of 1934, a chloride content averaging more than 300 ppm and reaching a maximum of more than 1 200 ppm.

FLOODS AND OTHER ACCIDENTS TO WATER SUPPLIES

Harrisburg, Pittsburgh, and Bellevue, Pa., Water Plants Flooded.—The spring and early summer of 1936 witnessed many floods in the North and Middle Atlantic States as well as those of the Ohio Valley. The water supplies of Harrisburg and Pittsburgh, Pa., as well as the water-works supplying McKees Rocks and Bellevue, Pa., were each out of commission for one or more days. At all these plants provisions had been made in the original construction for floods several feet higher than ever recorded. The Susquehanna River, at Harrisburg, the Allegheny River, at the Pittsburgh plant, and the Ohio River, at the Bellevue and McKees Rocks plants, rose to unpredicted heights, and in each case submerged the pumping stations. At Harrisburg, the filter plant, which was surrounded by a high dike, was also submerged.

Elk City, Okla., Dam Washed Out.—On May 1, 1936, an earth dam at Elk City, Okla., was overtopped during a rainfall of 4.7 in. in 2 hr. The estimated run-off was 16 000 cu ft per sec, or 680 cu ft per sec per sq mile, on about 23.5 sq miles. The original provisions were for 425 cu ft per sec per sq mile, but the crest of the spillway had unwisely been raised 2.5 ft to increase the reservoir capacity.

Overtopping of Dam on Little Wolf Creek, Illinois.—This dam was washed out in a flood from a 6-in. rainfall estimated to be about equal to the capacity of the spillway. This washing out was not witnessed, but was ascribed to a reduced free-board from settlement. This illustrates the necessity, known to good operators, of having levels run over the tops of earth dams from time to time and any settlements corrected by additional filling.

Foresight by James B. Francis Justified.—A huge gate, 29 ft long, 25 ft high, and 20 in. thick, suspended by a loop of 5 by $1\frac{1}{4}$-in. strap iron on a framework of huge hand-hewn timbers, was installed in 1848 on the power canal at Lowell, Mass., which leads from the Merrimac River. This gate was put in by the eminent hydraulic engineer, the late James Bicheno Francis, Past-President and Hon. M. Am. Soc. C. E., over the objections of the Directors of the Canal Company. It was first used four years after it was built (in the flood of 1862) and then fell into disuse, being called "Francis' folly." Once again, however, on March 19, 1936, it saved the City of Lowell from flood.

Rainfall and Run-Off Records in Colorado

An unusually fine report on record rainfall and run-off occurrences in Colorado has been presented by M. C. Hinderlider, M. Am. Soc. C. E.[3] Not only are the peak-flood quantities given, but also graphs of the entire flood run-offs, which are invaluable in the design of spillways and reservoirs for flood control. The Committee records its belief that there are too many unreliable records of flood-peak flows, and only a small fraction of the number needed of complete records of floods, that is, records such as those made public by Mr. Hinderlider. There should be many more continuously recording gages placed on large and small streams at places where reasonably reliable records might be obtained. Flood peaks estimated by the slope-and-area method (which are the sole reliance in many cases) frequently give unbelievable results, and spillways continue to be built which are entirely insufficient as far as peak records of water-sheds of similar areas are concerned, but without knowledge as to whether complete graphs of such floods would justify the smaller capacity.

Progress and Innovations in the Design and Construction of Dams

The Committee would note at the outset an interesting investigation by a special engineer in the State Engineer's Department, in California, which has extended over a period of six years and has included the investigation of about 900 dams. One-third of these structures were found to need repair. The classification as to repairs or alterations needed is under four headings: Inadequate spillways; inadequate treatment of foundation; lack of protection against erosion; and poor materials or poor construction methods.

Improved Control of Earth Dam Construction.—In the construction of earth dams the improved methods of grading materials, control of moisture to produce optimum compaction, and testing the degree of compactness, which were first put into systematic form during the construction of three earth dams several years ago for the water supply of Los Angeles, Calif., have gained ground among well-informed engineers. These methods have been adapted to the construction of other important dams such as: Dam No. 1, San Gabriel Canyon, which is being constructed by the Los Angeles City Flood Control authorities; the Cajalko Reservoir Dam at the terminus of the Colorado River Aqueduct, supplying the Metropolitan District of Los Angeles; the Sutherland Reservoir Dam, in Pratt Valley, Nebraska; the Public Power and Irrigation District, and other smaller, dams. The descriptions of the construction of the dams of the Muskingum project for flood control in Eastern Ohio also indicate the application of soil mechanics, but the Committee has not at hand the details as to the control used.

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[3] *Engineering News-Record*, August 13, 1936.

The Committee would remark that there is need of a careful evalution of results obtained by these improvements in comparison with those obtained by methods used by skillful dam builders of previous generations, who selected their material and governed its moisture through judgment based on experience and by observation of its behavior during rolling. The embankments of Ashokan Reservior (New York), for example, had tested weights of 130 lb per cu ft, or more, which was perhaps as good as practicable with the material, and at any rate was good enough with the generous sections used. Many other dams built by the "old masters" appear to have been perfectly successful in accomplishing the desired purpose. The complication of methods of control is doubtless often justified, but may frequently lead to unnecessary expense. The demonstration of a new method tends to put an element of fear into those who are not in a position to investigate or to use the rather complicated laboratory methods employed in the newer method; hence, an evaluation would be helpful and the Committee hopes some one will undertake it.

Rock-Fill Dams.—The progress reports of this Committee have not hitherto included reference to the lessons drawn from the bad settlement of Rock-Fill Dam No. 2, 265 ft high, on the San Gabriel River, in California. The trouble was diagnosed as insufficient filling between the fragments of the rock-fill; hence, in Dam No. 1 in this canyon, now under construction, the down-stream face of which is rock-fill, extensive flushing has been done so as to fill the voids of the rock-fill with fine material. Some engineers will recall that certain rock-fill embankments supporting the old Croton Aqueduct, built about the middle of the Nineteenth Century, settled badly and for many years rendered the aqueduct non-usable. Only in recent years have repairs been made. Since the Croton experience, aqueduct builders in the East have not placed aqueducts on rock-fills. With the flushing in of fine materials it is possible that subsequent settlement will be kept within narrow limits. Careful observations of settlement should be, and doubtless will be, made on these new structures to determine whether the problem has been solved.

Freezing for Support of Moving Ground at Dams.—Recent reports record the use of refrigerating pipes at the Grand Coulee Dam for the purpose of freezing a sliding mud-bank in position during the excavation for, and the construction of, the foundations.

Refrigerating Concrete in Masonry Dams.—Previous reports noted the complete and systematic cooling of the Boulder Dam by refrigerating pipes cast into the concrete masonry. It is interesting to note that a similar method is being used at the Grand Coulee Dam, except that in this latter case cold water, which is available, is being used instead of refrigerated brine water.

Precautions Against Earthquakes in Dam Construction.—Five storage dams are being built in the Santa Clara Valley, California, on water-deposited soils over the shaken bed-rock between the San Andreas and Haywood faults. These dams were on five streams each of which follows a fault or other weakness. It is the belief of the builders that these dams will withstand earthquake shocks as severe as any that have been experienced. The material is being carefully graded, using the most impervious material up stream and the most porous material down stream. The moisture content is being kept within 2% of the optimum for the densest fill. This optimum varies from 10% to 17%, depending on the character of the material. Sheepsfoot rollers are used. Cut-offs are made of earth; the thickness of the impervious parts of the dam, and the height of the free-board are generous in relation to the magnitude of the differential horizontal and vertical movements which seismographic study showed were possible along these faults. Down-stream and up-stream slopes are gentle (from 1 on 3 to 1 on 4). The slopes are covered with gravel and loose rock, rather than rigid concrete or stone paving, in order to provide a self-healing weight to keep the impervious material in place. The State Engineer, the Deputy State Engineer, and a board of other eminent consultants approved these designs as safe.

Theories of Masonry Dam Design.—The Committee wishes to emphasize again the need for a determination by the Engineering Profession of the conditions, if any, under which uplift water pressure and ice pressure are exerted on gravity masonry dams and the extent of such forces; also, for authoritative analyses of the stresses in such dams. A wide variation in design still exists and the safety of many dams may be questioned under theories now commonly used in designs of important dams.

Progress in the Design and Construction of Pipe Lines

The Metropolitan District of the Los Angeles Region is constructing for the Colorado Aqueduct, and the distribution lines from it, pipe lines of large sizes, including pre-cast reinforced concrete sections, 12 ft 8 in. in diameter, and welded steel pipe varying from 8 ft to 12 ft 8 in. in diameter. The latter are cleaned of all mill scale by automatic steel-dust air-blast. The interior is coated with coal-tar enamel spun on, and the outside with gunite.

Cathodic Protection of Steel Pipe Lines of East Bay Municipal District Near San Francisco, Calif.—Protection from galvanic action for the steel pipes forming part of the aqueduct which carries water from the Davis Dam on the Mokelume River to Oakland, Alameda, and other communities of the East Bay Municipal District, is being accomplished by motor-generator or rectifier sets, ten in number, which produce a controlled potential in all parts of the line. Before such protection was provided corrosion was caused by electric currents produced by the reaction of the soil and the steel pipe. The pipe line has a wrapped covering, but this failed, principally by cracking along the top center line, caused by soil stresses.

Soil Investigation in Connection with Corrosion of Pipe Lines.—The corrosive nature of the soil in certain Western States, notably California, has for many years stimulated investigation of the soil along proposed pipe-line routes, at first those for oil and gas, but more recently those for water. The so-called Shepard rod (which is pushed into the ground at various depths and by means of electric terminals and an Ohm meter measures the electrical resistance of the ground), is a commonly used instrument. The Los Angeles Bureau of Water, for example, uses it extensively. A low resistance indicates corrosive ground and by correlation with observations on the condition of metal pipes the nature of the ground at any given locality can be classified as severely, moderately, or only mildly, corrosive.

At the Los Angeles Water Bureau Laboratory tests of soil are also frequently made. Nipples of $\frac{3}{4}$-in. steel pipe, 4 in. long, are embedded in the earth to be tested. The earth is moistened and is used as the electrolite through which a direct current of 6 volts flows from nipple to containing can. The degree of corrosion in a given time is a measure of the soil which may be correlated with observations on the pipe.

Corrosion-Resisting Coatings for Pipe.—Also Treatment of Water.—The trend toward protection of carrying capacity is becoming accelerated, as evidenced by the increased interest in water-works meetings, and the increased use of cement linings and spun refined tar linings. The idea that one buys capacity when purchasing pipe lines has been long in percolating into the intelligence of water-supply engineers and superintendents; but progress during the past year or two has been encouraging. More attention is given to the treatment of water to prevent rapid deterioration of existing tar-coated mains.

Water-Hammer in Pipe Lines.—S. Logan Kerr, M. Am. Soc. C. E., Chairman of the Water Hammer Committee of the American Society of Mechanical Engineers, has kindly furnished information on this subject. He states that in the past few years a new tool has been available to engineers concerned with problems arising from water-hammer; namely, the graphical method of solution. The involved computations previously required have been greatly simplified, and it is now possible to determine accurately the surge conditions occurring when a motor-driven pumping unit suddenly stops and the check-valve closes. The hazards of branch pipes and dead ends can also be studied readily.

The A. S. M. E. Committee on Water Hammer, with the co-operation of the Society, the American Water Works Association, and others, has definitely scheduled its "Second Symposium on Water Hammer" for the first week in December, 1937, in New York City. At this meeting a comprehensive review of the status of the water-hammer theory will be presented, and several papers will be devoted to the description of the new graphical methods and will illustrate their application to cases involving water power plants, and other problems.

Mr. Kerr calls attention to the more general recognition of the advantages of variable rates of closure for valves; that is, starting at a rapid rate and completing slowly. The Committee notes that this matter was extensively studied about 1910 by the New York Board of Water Supply and applied to the 48-in. and 72-in. riser valves of the City Tunnel. Other valves designed in recent years embody the principle.

PROGRESS IN THE DESIGN OF PUMPS

The increase in general efficiency of centrifugal pumps, both of the ordinary type and of the various well types, continues and is being extended to small sizes in a manner previously thought unlikely. Well pumps with capacities as small as 250 gal per min may now be obtained with efficiencies exceeding 75 per cent. The Committee must not omit reference to the very efficient design for centrifugal pumps produced for the Colorado Aqueduct by co-operative effort of the pump manufacturers and experts employed by the Metropolitan Water District of Southern California. In the course of these studies there were developed and used, at the California Institute of Technology, Pasadena, Calif., extraordinarily complete and precise instruments for testing in models the effect of each element of design, a special laboratory being constructed for the purpose.

The pumping problem called for two stations with a pump lift of about 440 ft, two stations with a lift of about 300 ft, and one station with a lift of about 150 ft. Large quantities of water will have to be pumped (about 390 mgd for the initial installation, and about 1 000 mgd when finally completed). A difference in operating efficiency of 1% on all the proposed pumps would correspond to a gain or loss of $40 000 per yr in power costs. The greatest mechanical reliability has been sought, as well as high efficiency. A new form of pump-casing for the higher heads was developed, resembling a short barrel closed by unusually strong covers, resulting in a very rigid design to prevent distortion of the casing, which might have proved fatal in view of the unusually close running clearances inside the pump. Tests of the models indicate an efficiency of 92% for the full-sized units.

PROGRESS IN THE TREATMENT OF WATER

The trend in the design of mechanical mixing basins has been in the direction of the flocculator type. Few mixing basins of the gravity-flow type are now being designed. Most of the designing engineers now appreciate the importance of forming an excellent coagulation so that there may be good settlement of the coagulated matter before the water goes to the filters.

There has been little change in the design of settling basins within the past several years. The two-story basins at Milwaukee, Wis., are a departure from the usual type of plain settling basin. Some consideration is being given to means of producing more rapid settlement of coagulated matter, and although it is known that there are means by which this may be done, the cost usually is not justified.

Almost no change has occurred in the design of filters within the past year. A new method of determining the effective size of sand has been proposed. Attention is being directed to the importance of maintaining filter beds in good condition. A few of the filter-plant operators now make occasional measurements of the volume of mud-balls in the filters as a guide to the efficiency of the washing facilities for keeping the beds in good condition. This probably is the beginning of better maintenance of filter beds.

Attempts to improve the design of filter bottoms continue, and several filters with porous plate bottoms have been constructed in various filtration plants. Use of such bottoms on an extended scale will not occur until the results on these experimental units are known.

There has been almost no change in the chemical treatment of water, except probably a slight drift to the use of ferric iron for coagulation. Silicon in a certain form has been found to give aid to coagulation, but it is too early to predict the extent of its use in treatment plants. The addition of magnesium sulfate to water and the precipitation of the magnesium as Mg_3 OH has been found to reduce the fluoride content of water.

Progress in the elimination of taste and odor in water has continued throughout the year, the chief advancement being in the extension of the use of powdered, activated carbon. With few exceptions, the taste and odor may be removed to a point where it is not objectionable, at a cost well within the amount which may be spent for such removal. The granular activated carbon units at Oshkosh, Wis., have been completed.

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January 21, 1937.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

SURFACE AND SUB-SURFACE INVESTIGATIONS, QUABBIN DAMS AND AQUEDUCT

A SYMPOSIUM

Discussion

By Messrs. Verne Gongwer, and Stanley M. Dore

Verne Gongwer,[18] M. Am. Soc. C. E. (by letter).[18a]—In the "Synopsis" of Mr. Dore's very interesting and valuable paper, two general sources of information for use in determining the permeability of earth over-burden are mentioned: (1) Dry samples from bore-hole investigations; and (2) the effect of pumping upon the ground-water conditions in the over-burden.

Mr. Dore has described a very careful and almost exhaustive application of these methods to the study of conditions at Quabbin Reservoir, and is to be commended for so thoroughly presenting it in his paper, because the characteristics of the underlying formations, where a dam is to be built on earth foundations, warrants the gravest study. This phase of the problem is too often given little or only perfunctory attention, usually being confined to the drilling of a few holes, sinking of a few test pits, and possibly doing a little perfunctory pumping from them; or confined to the application of some well known formula to a conjectural distance for seepage or "creep" by means of an effective size determined in the laboratory from a greater or less number of usually questionable samples recovered from the explorations. The formulas seldom have been developed for the conditions prevailing at the site under investigation, and the results of laboratory analyses and tests can be no better than the samples submitted to the technicians; in fact, the usual unwitting inaccuracies in the recovering of samples, even where great care is intended, often render the laboratory results seriously misleading.

The writer fully agrees with Mr. Dore's "Conclusions," and desires, for emphasis, to quote from them:

> "There seems to be a scarcity of data concerning the permeable characteristics of soils and there are few, if any, definite methods in common use

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Note.—This Symposium by Frank E. Winsor, M. Am. Soc. C. E., and Stanley M. Dore and Frank E. Fahlquist, Assoc. Members, Am. Soc. C. E., was published in March, 1936, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings* as follows: August, 1936, by Messrs. Bayard F. Snow, and Ole Singstad.

[18] Project Engr., Guntersville Dam, TVA, Guntersville Dam, Ala.

[18a] Received by the Secretary February 25, 1937.

for making satisfactory determinations of the permeability of an overburden. In many cases permeable qualities of foundations have been a matter of extremely rough estimate, of guess, or merely of conjecture."

Very pertinent also is a comment made to the writer by E. W. Lane, M. Am. Soc. C. E., that engineers have advanced greatly in their study of soils and the development of methods for building excellent and water-tight earth-fill dams; but that too little attention is paid to the materials and foundations upon which such structures are built and that they are woefully lacking in means and methods of attack.

The writer has found attempts to determine the permeability of most soils (other than uniform silts, clay, and fine sand), from samples, to be very unsatisfactory and misleading. In fact, they can be said to be almost invariably misleading, due to the difficulty of recovering, in place, truly representative samples of streamers, lenses, or veins of the more open gravels without admixture from adjoining layers of materials, above or below. This condition vitiates the accuracy of either mechanical analysis curves or direct testing of "so-called dry samples" for permeability.

Observations of the variations in, and effect on, the ground-water levels from pumping from open test pits, casings, or well-points is believed to furnish much more illuminating and dependable data in regard to permeability; however, the writer has never been able to arrive by this method at anything very tangible, other than a general indication of relative water-tightness. There have been certain surprising results, in that test pits in areas previously considered quite tight from borings and mechanical analyses of samples, have produced water under the pumping test almost as freely as other pits that have been expected from mechanical analyses of samples to produce water freely. In this connection, the writer's experience coincides remarkably with certain conclusions by Mr. Dore; for example, in the making of certain calculations of the quantity of water which would flow horizontally under a given head through a vertical 1-ft slice of the materials found in a test pit from surface to bed-rock, it was found that, theoretically, on the basis of flow constants corresponding to results of mechanical analyses, from 90% to 95% of the total water passing will be transmitted through, or by, the coarsest stratum penetrated by the test pit, even if that stratum is relatively insignificant in thickness and the "effective size" of the particles is not particularly, or startlingly, large. This lends considerable importance to a conclusion by Mr. Dore, which is also heartily concurred in by the writer, that although the coarser materials are found or are usually considered as lying in random assortment or lenses, they are usually in glacial soils—and almost invariably in alluvial soils—either connected or continuous in the general direction of the valley, or should be considered continuous for safety.

When a dam is proposed to rest on earth foundations, with or without underlying rock at shallow or greater depths, two main questions arise, which must be answered in advance:

(1) Will the volume of the total leakage through the sub-strata be excessive from the standpoint of loss of water, unsightliness, possible dam-

age to property, or for any of a number of possible reasons, other than as affecting the safety of the dam itself?

(2) Are the materials, or disposition of the sub-strata, such as to render at all possible the disastrous phenomenon resulting from concentrated flow at one or more points, at high velocity, known as "piping"?

A positive answer to Question (1) or to both questions, of course, will call for a positive cut-off. This may apply either to the over-burden or to the underlying rock, where it is cavernous or seamy. In some few cases it may be permissible to "take a chance" and determine the answer to Question (2) by constructing the dam and awaiting results, trusting that correction will be then possible, but in the vast majority of cases too much is at stake.

From Mr. Dore's paper it is assumed that, at the Quabbin Dams, the preliminary data and investigation somewhat indicated the need of a positive cut-off, and that the first caissons constructed, although located and designed for possible incorporation in a cut-off wall, were by way of pioneering for positive proof that the completion of such a cut-off was actually necessary. The analysis of results of subsequent pumping from these caissons and sumps furnished a definite answer to Question (1) but not necessarily to Question (2), upon which the need of a cut-off could be decided. It is usually difficult to provide such an ideal pumping arrangement as that described by Mr. Dore except where caissons have been installed as at Quabbin, and large capacity pumps are usually necessary.

With the cut-off once decided upon, and assuming that it will be tight in itself, tightly sealed upon the rock, and that the rock itself is tight, an answer to Question (2) is not essential. However, since extreme difficulty of construction or other previously unknown hazards sometimes leaves doubt as to the existence of possible flaws in the cut-off, or in the seal to the rock, and due to the difficulty or cost of detection and correction, an answer to Question (2) is very desirable, both before and after the construction of the cut-off. This would scarcely be necessary with the type of cut-off adopted at Quabbin, but it is sometimes the case where a steel sheet-pile cut-off is adopted, or even more positive and seemingly thorough means are used, as is attested by comparatively recent occurrences at the Ontelaunee Dam, at Reading, Pa.

As stated, although the writer, like Mr. Dore, considers the observations and data obtained as the results of pumping in pits and test wells of considerable value in determining the average effective permeable quality of the over-burden as a whole, where this factor is desired, he does not consider the average permeability of an over-burden (which is to become the foundation material of a dam) of any great value. It may even be disastrously misleading, if considered alone, in determining the safety of the dam from "piping" or other phenomena of undermining, because calculations of velocities of flow of under-seepage from average permeability will result, ordinarily, in a very re-assuring figure for velocity. The maximum rate of flow through the coarsest stratum, lense, or vein of material existing at the site, will indicate the possible danger from piping and if determinable will be vastly greater and considerably less re-assuring. Such conditions will seldom be discovered by pumping tests or by any known method of sampling, except from test pits, which are usually much too far apart for this purpose. Let it be assumed that the vein in question has no actual direct connection to the river or watercourse, or to the ground surface under the proposed reservoir by any of a large variety of possible means. Then, under full reservoir conditions, all other materials of the valley floor or flood plain are acting, or will act, as a vast filter admitting water to the vein rather freely and in considerable quantities, producing a high velocity. Should the vein lie relatively close to the surface of the reservoir floor, or have connections close to it, the so-called piping distance necessary to protect against failure is reduced from a comfortably long distance greater than the base width of the dam, to substantially no more than the vertical depth of the over-burden above the seam. In most cases the distance is reduced to the thickness of a fairly thin stratum of comparatively impervious material, which cannot usually be proved to be continuous or without fault or opening. It is immaterial, of course, whether the pervious stratum is of coarse gravel or whether it is an open and uncorrected seam or solution channel in the underlying bed-rock. In either case the relative conducting capacity is so great under the increased head of the reservoir as to warrant deducting practically the entire width of the base of the dam from any calculation of safety from "piping."

Whether or not one or more sink-holes form in the bottom of the reservoir the area of the filter bed will usually be such that water will be supplied to any open vein or stratum with so little loss of head that although the "blanket" below the dam is thick and comparatively impermeable, the hydraulic gradient of the seeping water, under the conditions, is likely to be well above the ground surface at one or more points. In some cases a pressure can be calculated readily to exist, which is sufficient actually to lift the overlying blanket, particularly if only one of several permeable veins should chance to end, or be substantially reduced in carrying capacity near the down-stream toe of the embankment. However, some spring or other natural "seep" will probably be present to encourage escape and to start "piping" action. The phenomenon of flotation of the finer particles, of course, will render the thickness and tightness of the down-stream blanket of little value, whereas the gravel seam or solution channel in the rock (the chances being that there will be a number of one or the other), will usually have ample unfilled voids to accommodate the displacement of the relatively small quantity of the "fines" from the up-stream blanket necessary to begin the action of piping at that end. The next step is the formation of sink-holes, and then, possibly, disaster.

The foregoing may seem a pessimistic view, but that such results are very possible is attested by the comparatively recent partial failure of the Ontelaunee Dam at Reading, Pa., previously referred to and recently described in engineering periodicals. The writer has often had opportunity to verify suspicion by excavating in foundations, and finding seams of large, perfectly clean gravel located in apparently quite impervious fine sand and silt, which seams were producing water copiously. He also has discovered numerous partly filled and water-bearing solution channels in limestone where few were indicated by the customary spacing of exploratory drill holes. Some "chimneys" were actually found leading into the latter, and subsequent pumping from the seams proved the presence of other "chimneys."

Fortunately, in the case of most important projects, some means of investigating the possible under-flow has been undertaken, and although not usually conducted with the thoroughness of that described by Mr. Dore, has resulted in a cut-off wall or other positive element being incorporated in the structure as a safeguard. Such action is seldom too conservative; in effect, it corresponds to the usual safety factor against uplift, sliding, etc., for concrete dams. The relative proportion of the cost of safety against under-failure of earth dams to total cost seldom, if ever, approaches that of the usual practice in utilizing three or four times the theoretically necessary material in wood, steel, or concrete structures.

Dam failures of recent years are believed to indicate that, through continued success, the profession is in danger of becoming somewhat too optimistic in regard to factors of safety, particularly in foundations with respect to the elements herein discussed.

The determination of permeabilities by laboratory analyses of bore samples is of value only in the case of the finer and more cohesive materials, as the result depends entirely upon whether a representative actual sample is furnished to the laboratory. Wash-boring samples are practically valueless. Samples of fine cohesive materials may be taken from the ends of drill casings by various methods, which will be suitable for mechanical analysis, but seldom in natural unconsolidated, or unbroken, state for direct permeability tests. True samples of the coarser veins, accidentally encountered, cannot be taken by any means or tool known to the writer, without the probability of mixing in it at least 10% or 15% of the finer strata above or below it, which admixture will control the "effective size" and render worse than valueless any calculations made from it. Such calculations may lead to serious errors in resulting decisions, always tending to the unsafe side.

This admixture of fines is almost certain to occur in sampling where thin strata of very open materials are encountered, even in open test pits, and particularly with running ground-water, unless a competent observer is present in the pit at all times. to take true samples. Samples from the bottoms of casings, below ground-water level, or where wash-boring methods are used, are always subject to question. In all cases where pumping has been resorted to in a casing or pit, samples of coarser materials may have fines washed into or out of them before the casing or pit has quite reached such a stratum. Furthermore, it is extremely unlikely that the coarsest stratum or other opening has been discovered unless bore-holes or test pits are prohibitive in number (at least as close as 10-ft centers over the entire area under investigation) and more unlikely that it has been identified and a true sample taken.

It is noted that Mr. Dore dissents from the use of the coarsest particle of the finest 10% of the sample or the term, "effective size," in making permeability computations, by means of the Slichter formula. After discarding certain extreme variations, such as 5% and 26%, however, he finds that the permeability is controlled in most cases by sizes within the fairly close range of from 8% to 15%, with a reasonable mean of all results at 12%, which he uses in connection with the Slichter formula. The writer agrees with his conclusion that all such methods are subject to errors of from 50% to 100%, and, therefore, such a small difference in practice in the selection of the controlling size seems unimportant. For that reason no close decision as to the need of a cut-off wall should ever be decided in the negative, on the basis of pumping tests or permeability calculations from samples, but a factor should be used, on the side of safety, considerably greater than the possible error inherent in the methods of investigation.

The belief and hope that the methods developed and used at Quabbin may find application elsewhere, has been expressed by Mr. Dore, and his paper adds greatly to the practical application of the meager methods and tools that are available for predicting under-seepage in order to determine the necessity of cut-off provisions. Where, as at Quabbin, pumping was done on such a scale, and for a period long enough so that the ground-water was considerably lowered along the axis of the dam from one abutment to the other, many valuable data and determinations are possible, particularly as to general permeability. However, high productivity in the caissons used as wells would seem to leave little doubt as to the need for cut-off provisions, as without such provisions it would seem probable that under full reservoir head the leakage would be at least as much as, or several times, the total rate of pumping.

In his investigations, Mr. Dore represented the variation of the ground-water table by means of contours. The use of ground-water contours is worthy and capable of wide general application to the study of sub-strata with respect to permeability and the writer has used this tool with satisfactory results at several locations in both glacial and alluvial soils. In general, the method involved a system or grid of a greater or lesser number of observation wells, consisting of auger holes, perforated pipes, supplemented by such test pits, existing dug wells, or ponds, depending on which were available With the latter, of course, care should be exercised to determine that the free water surface does not represent a perched water-table, as frequently happens in the case of ponds, ditches, and even rivers at certain locations. Periodic readings of the elevation of the water in such wells are taken very conveniently and accurately with a 50-ft or a 100-ft steel tape (with 0.01-ft graduations) on a compact reel, and fitted with a cupped lead weight.

By taking observations of all wells and plotting ground-water contour maps at 1-day, 5-day, or 10-day intervals, during variations in the water-table, much may be deduced in regard to the direction of flow of the underground water. Occasionally, different elevations will be registered in pairs of pipes driven to different depths, where the ground-water is moving rapidly in some stratum. The variations in the water-table may be due to the natural rise and fall of the adjacent reservoir or river; or it may be produced by "gunning" the river by means of discharge from the outlet valves or spillway of some dam, or by pumping from various pits or wells. The time lag and the response of the wells to fluctuations in river or reservoir level, will give a general indication of the relative general permeability of the materials. It is very enlightening to plat time graphs of a group of wells, superimposed upon a time graph of the river-gage readings.

Certain general information may be derived from the displacement of the ground-water-table in a given time, over a given area, due to a rise or fall of the river; but usually a calculation from the amount of upward displacement of the water-table is of no value on account of the impracticability of calculating the quantity of ground-water escaping from the area in question. Valleys in the water-table contour maps will often indicate this condition and the direction of escape.

The ground-water contour maps would be closely analogous to contour maps of the free water surface of a flowing river were it not for the varying resistance of the materials through which the ground-water is moving. A steep slope or a nearly flat slope in the water-table may indicate various rates of flow, depending upon the characteristics of the materials. The difficulties in determining the characteristics of the more open materials by sampling has been discussed. For a complete and dependable determination of conditions, therefore, it is necessary to have a means for determining directly the actual rate of flow in the soil at definite locations and depths. Such a means was happily suggested to the writer by chancing upon a reference to the electrical ground-water timing apparatus devised by Professor Charles Slichter.[19] Such an apparatus was constructed for use at one dam site, with the exception that the timing and recording were done by attaching a commutator to the shaft of a recording ammeter of the type used on power-house switchboards.

The configurations of the water-table contours will usually indicate valleys where, presumably, the rate of underground flow is greatest. In the first two such locations selected, in a terrain previously considered quite impervious, rates of flow approximating 400 ft per day were observed. Sal ammoniac was used as the electrolyte, and the direction of flow checked that assumed from the contours. At one place it was at right angles to the river and at the other location approximately parallel to the river, both being caused by the same rise in the river level. The direction was corroborated further by the fact that the only electrode affected in each case, being brass, was found upon removal to have been discolored to a dee purple, whereas the other electrodes were still bright. The discoloratio was easily removed by sand paper after each test. Calculating backwar from the measured rate of flow indicated an effective size of the pa ticular gravel vein approximately ten times larger than that obtaine from the coarsest samples recovered from 24-in. casings driven to roc at several places relatively near-by.

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[19] *Ground-Water Paper No. 140*, U. S. Geological Survey.

With a few determinations made in critical or representative location and with the slope of the water-table accurately determined by precis measurements in the wells used with the Slichter apparatus, and roughl checked from the contour map, a much more complete understanding c conditions is had than may be determined by any other method known t the writer. If the hydraulic gradient found between the test wells differ materially from the general slope taken from the contour map, the latte should be disregarded in calculating the flow constant.

Remembering that even where the materials do not differ greatly i gradation (approximately 90% of the total water movement will be foun to occur in the one coarsest layer), the pipes for the Slichter test ma be perforated for only 2 ft or 3 ft at the bottom and driven directly t this stratum when its location is known. Where the locations of suspecte coarse strata are unknown, the pipes may be perforated for the entir depth of the over-burden if desired, to permit a maximum flow rate bein recorded on the first trial. On the other hand, 2 ft or 3 ft may be per forated, and the pervious strata located, by surveying the entire depth o over-burden by successive tests, driving the pipes deeper each time by distance equal to the length of perforations. The pipes may be driven b hand for shallow depths, but where the over-burden is deep or gravelly a drill rig of the usual type will be found to expedite the work greatly.

If successive tests are to be made at the same set-up of pipes, it wil usually be found necessary to allow from one to several days between test for the sal ammoniac or salt from the previous test to become dissipate or washed away.

This direct method of measuring the flow through the actual material in place, at sites selected by means of the ground-water maps, seems fa superior to calculations from soil samples. It is not particularly neces sary to compute the effective size of grains from the Slichter test, a this apparatus provides a direct measurement of rate of flow under a know hydraulic gradient, and the possibility of piping under increased gradient may be calculated directly from it.

A word of warning may be in order in regard to the use of this equip ment in locations where solution cavities or other water passages or fault are possible in the underlying bed-rock. Assuming that the pipes have bee properly cleaned, and unless the materials are entirely clay or fine silt should the apparatus fail to register a positive test after a reasonable tim interval, it is entirely possible that the flow may be straight downward or i an inclined direction beneath the receiving electrodes. The ground-wate map, if constructed from a sufficient number of test wells, should be examined carefully for the presence of a depression or pot-hole, marked by a closed contour over a small area, which may not have been recognized in the interpolation of the contours. This condition would indicate a "chimney" in the rock, into which, under certain conditions, the ground-water may be flowing. It is often desirable to drive additional test wells to assist in solving peculiar conditions.

To avoid disappointment, and to attain satisfactory results in the use of driven perforated pipes for observing ground-water fluctuations and for use with the Slichter apparatus, the following suggestions are offered. The writer has found that the commercial type of screened well-points are not sturdy enough to stand the hard driving necessary in deep, sandy, or gravelly over-burden. The most satisfactory and economical points were devised by blowing four rows of holes at 90° in extra heavy pipe, the holes being spaced 4 to 6 in. in the rows, with the oxy-acetylene torch, and pointing and welding the ends of the pipes in the same manner, or by employing a blacksmith. The pipes should be 2 in. or larger, as desired or required.

After driving, the pipes are frequently found to be plugged with sand or silt forced through the holes, even if the water level may seem to stand at approximately the correct height. The pipes should be washed by inserting a $\frac{1}{2}$-in. or $\frac{3}{4}$-in. pipe used as a water-jet, rather than by attempting to blow or jet them clear with air. After the air pressure has been turned off the inrushing water usually plugs the pipes again with silt and sand, and serious error and lost records result in assuming that they are clean. The writer has seen pipes stand inactive for days after several attempts to clean them with air, and later register freely after washing them with a water-jet. The pipe lengths should be measured as driven, and the sounding lead used to ascertain whether they are clear for the full length after the completion of driving. After driving and cleaning, if necessary, each pipe should be tested by pouring a bucket or two of water into it and observing with sounding lead whether the water level subsequently subsides at a reasonable rate. Only if the perforations are in contact with very dense silt or clay will the water level remain stationary. When tested with the bucket, all stationary or sluggish pipe should be regarded with suspicion.

In driving, all pipes should be threaded so that, with application of considerable force, the ends can be made to abut firmly in the couplings. Ordinarily, extra heavy pipe should be used, with recessed couplings.

Driven pipes are seldom exactly plumb and may deviate several feet. For this reason, pipes driven for the Slichter apparatus should be spaced well apart to reduce the percentage of possible error in spacing between the perforated parts, and the corresponding error in the apparent timing of passage of the electrolyte. A spacing of 20 ft to 25 ft from the salted well to the receiving wells is desirable, although this will require more time for the observations, and additional receiving pipes and electrodes may also be required.

Considerable difficulty and annoyance will be avoided if the perforated basket for lowering the dry sal ammoniac, the insulators for the electrodes, and sounding leads used on steel tapes are all made sufficiently small to avoid catching on the burrs on the ends of the pipes in the couplings.

Clothes-line ropes and electrical wires should be very securely attached to the electrodes, and the wires protected from injury, at such connections, by electrician's tape. Measuring marks should be established on the ropes or wires to ascertain that the electrodes reach the bottom of the well. The electrodes should be lowered slowly and carefully to place, and thereafter should not be pumped up and down as the pumping action of the insulators has been found to draw sand into the pipes, which usually means jammed and lost electrodes and a useless pipe.

Apparently, for the purpose of being able to utilize a recording instrument, the Slichter apparatus is arranged for the use of dry cells, the measured variations in the induced current, when connected to the several circuits, furnishing the desired information as to the passage and arrival of the electrolyte. If the automatic-recording feature is not desired or essential, the process may be reversed and the varying resistance in the several circuits measured at regular intervals by means of an electrical "megger" or ground tester. At one time the writer utilized this principle with fair success between observation wells spaced approximately on 100-ft centers but without electrodes, having previously charged an up-stream well with common salt. The "megger" has certain advantages over the recording ammeter since difficulties are frequently experienced with the latter in poor commutator contacts and in adjusting the number of dry cells so that the record will stay on the scale of the paper. Since the resistance is very low between the electrodes and their respective pipes after the electrolyte has entered the pipes, it is probable that two or three dry cells and a small, comparatively insensitive ammeter, carried from pipe to pipe, would give satisfactory results, and much wiring and labor in setting up would be obviated. With this method, after the electrolyte has definitely reached one of the electrodes, the observations could be taken more frequently if desired, and observations on the other wells omitted, which is not practicable where the commutator clock is used, and the observations timed to 10-min or 15-min intervals, since there are eight different circuits in the regular set-up.

Where a grid or system of observation pipes has been driven for the study of a site, all pipes possible should be preserved in order to continue the program of periodic readings throughout the filling stage of the reservoir and for some time thereafter. Should it not be convenient (as in the case of a rolled fill) to extend certain of the pipes up through the fill to the crest of the dam, it is believed to be a justifiable expense, particularly where a cut-off has been constructed, subsequently, to drive a sufficient number of pipes through the completed fill into the more pervious strata of the foundation, closely up stream and down stream of the axis, in order to observe the hydraulic gradient at different locations after the filling of the reservoir, and to check the efficiency of the cut-off provisions.

In addition to the Slichter apparatus, valuable data have sometimes been obtained by the introduction of fluorescein dye into the perforated test walls and timing its arrival in test pits and excavations in which pumping is being done. In one case a rate of flow of approximately 200 ft in 2 hr was observed on a moderate gradient, in a limestone seam, with the probability that the greater part of the 2 hr was consumed by the fluorescein finding its way from the well into the chimney leading into the seam.

STANLEY M. DORE,[20] M. AM. SOC. C. E. (by letter).[20a]—The discussion by Mr. Snow calls attention to the "more or less blind groping" that occurs in the investigation of the permeable qualities of an over-burden which depends solely upon analyses of bore-hole samples. The writer agrees that the pumping of ground-water from one of many measuring wells is decidedly, and by far, the superior means of estimating the permeability, and should preferably be used wherever such means are available. However, in many cases, the determination might be made after the bore-hole casings have been pulled. For such cases the usefulness of results obtained by analyses of bore-hole samples depends upon the number of samples available and the manner in which the volume of the over-burden has been investigated with bore holes. To take samples from a relatively small number of bore holes for the determination of the permeability of an over-burden as a whole is practically useless, as the result obtained may be correct or it may be absurd, and in such a case the groping is decidedly blind. However, with a large number of samples from an adequate number of well-placed bore holes, the law of averages operates to reduce the guesswork and materially to increase the probability that the result is a reasonable one.

For free-draining glacial materials, similar to those at the sites of the Quabbin Dam and Dike, it was estimated that about 90% of the water in the pores was obtained quickly from storage by pumping. Probably, having been under water for years, the pores of these materials were entirely filled with water. Part of the remaining 10% would trickle slowly down to the lowered ground-water surface, and part would be held in the corners of the pores and would never travel toward the pumps.

The division of the quantity pumped into that obtained from depletion of storage, that obtained from infiltration within the circle of influence, and that obtained from flow into the circle from without, can not be precisely made for any site for all periods of time, as it depends upon variable conditions, such as rainfall, temperature, run-off, vegetation, etc. The quantity flowing underground into the circle from without is usually relatively very small; and, for a valley-shaped over-burden, it is the quantity that ordinarily flows down the valley underground. For instance, for the relatively pervious over-burden at the Quabbin Dike such quantity was estimated to be about 100 gal per min. The quantity flowing on the surface into the circle from without depends on local conditions, and the quantity that reaches the areas affected by the pumping operations depends upon the slope and water-tightness of the stream bed. At Quabbin Dike nearly all the surface drainage upstream (drainage area, 1.4 sq miles), flowed indirectly into the circle of influence, sank into the ground, and was pumped. Practically, then, the water pumped is nearly all infiltration within the circle of influence and depletion of storage. The quantity of pumpage remaining constant, the quantity depleted from storage, depends directly upon the infiltration, which in turn, is large or small as affected by recent rains or other conditions. At the Quabbin Dike the percentage of the total pumpage coming from depletion of storage varied generally from about 10 to 90.

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[20] Associate Civ. Engr., Massachusetts Met. Dist. Water Supply Comm., Boston, Mass.

[20a] Received by the Secretary February 15, 1937.

Mr. Snow expressed a desire that the determinations of permeability might be used as means of estimating the yield of ground-water wells. It was the writer's intention that such would be the case although he presented the investigations in connection with unwatering and seepage problems. In both cases the permeability of the over-burden would be estimated in a like manner the data in case of a water-supply development being obtained, if possible by test pumping from one bore hole and observing the drop in water level in others.

Once this average permeability is obtained, circles of influence can be estimated, and the ground-water maximum draw-down curves plotted in profile for wells at various depths. The maximum continuous yield of any well after the ground-water curve is drawn down to its maximum and there is no further reduction of storage available, depends upon the infiltration portion of the pumpage, which in turn depends upon local conditions of rainfall and run-off and of surface and sub-surface flow into the circle of influence. This part of the yield as a continuous water supply is comparable to the prime power of a hydro-electric development, and in case of an extended and continuous excessive demand, the yield of the well will be limited to this quantity. However, the demand upon a well as a water supply is seldom constant and continuous. The volume of the pores between this maximum draw-down curve and the original ground-water surface, in such a case, can be utilized as a storage reservoir, to be filled in wet periods and drawn upon in dry periods.

Thus, the underground supply can be treated in a manner similar to the surface supply, the infiltration being the source and the volume of pores above the maximum draw-down curve being the storage reservoir, upon which certain demands can be made, limited by the capacity of the pumps and the permeability of the over-burden. For estimating the maximum draw-down curve and the rate at which water can be drawn from the over-burden for other positions of the draw-down curve, the same formulas for pumpage as were described in the paper apply when the permeability is known.

After the local factors affecting the infiltration, the volume of underground storage, and the rates of pumpage permissible at various depths have been determined, the probable capacity of any well to meet certain demands can then be estimated. The spacing of wells is most advantageous if the circles of influence do not interfere, but, in such cases, although more complicated, the yields may be similarly estimated.

Mr. Gongwer evidently feels that, although the methods described in the paper are useful in obtaining the average permeability of an earth over-burden, such a value does not necessarily indicate the lack of a need for a cut-off in case this result is a relatively small one, as the average permeability is not indicative of what the largest permeability of some of the materials may be. Of course, if the total quantity of percolation indicated by the average permeability determinations is large enough to warrant the construction of a cut-off (which was the case at the Quabbin Dams), the distribution of this percolation throughout the over-burden is not important, except in cases where the cut-off provisions considered are partial or not positive. The writer agrees with Mr. Gongwer that, for the partial or non-positive cut-off provisions, and for the case in which the total quantity of percolation in itself is not large enough to warrant a cut-off, the final decision on omitting the complete cut-off provisions should not be made until investigations and studies indicate that there is no serious concentration of the total percolation (although small) in one or more veins, channels, or strata, which might jeopardize the safety of the dam through "piping." For such cases, the presence of a concentrated flow condition would be definitely indicated in a pumping program with an adequate number of observation wells, because the loss of head in the wells through or near the coarse veins or strata would be comparatively much less than elsewhere at the site where the wells were in the tighter materials. The writer, however, wishes to thank Mr. Gongwer for describing methods used in determining the location and size of concentrated flows in over-burdens. These constitute a valuable addition to the paper as they can be utilized, if desired, in connection with a pumping program or with bore-hole investigations for locating and measuring concentrated flow conditions or for demonstrating the lack of any such flow.

Mr. Gongwer indicates that it is practically impossible to obtain satisfactory bore-hole samples that contain the proper quantity and quality of coarse materials to indicate the existence of the coarser veins and strata in the over-burden. For certain types of soils, such as alluvial deposits, the writer agrees that this is the case, but in many soils, such as random glacial deposits, reasonably good, so-called, dry samples are obtainable from bore holes which are fairly representative of the materials from which they are taken, the coarse materials usually occurring in thick veins, pockets, lenses, and strata. If the number of samples taken is large enough to be indicative, the chances of appreciable distortion in the estimate are not large, although the writer agrees that, as in the case of the pumping program, the possible existence of any coarse "piping" channel should be investigated before definite conclusions are reached regarding the omission of the cut-off.

The writer notes particularly that Mr. Gongwer concurs with him in the opinion that in over-burdens the coarser veins or pockets are usually interconnected, because the study of permeable qualities of an over-burden, especially from bore-hole samples, cannot be made reasonably, if it is assumed that the coarser portions are segregated and insulated from each other by the finer parts.

It is believed that, although the pumping arrangements at Quabbin are fairly ideal, because of the caisson pumping intakes, it would not be difficult to obtain a good pumping program elsewhere using well-points and pumps installed in the bottom of small pipe wells sunk to the desired depths at suitable locations.

Mr. Gongwer's comparison of the expense of cut-off provisions with the cost of the materials used to obtain the factor of safety in structural work is particularly well made; but sufficient and proper investigations in some cases will reveal that considerable saving can be made instead of spending money for cut-off insurance, and, in other cases, will eliminate the practice too often followed of deciding not to build a cut-off, risking the unknown consequences.

The point that the writer wishes to emphasize is not that the 12% size was a reasonable means to be used in making permeability computations by means of the Slichter formula for materials at Quabbin Dams, but that it might be some size other than the 10% size commonly used for the materials found at some other location, say, the 2%, the 5% or the 15% size; and that in using the common method the permeability results might be distorted so seriously as to be extremely misleading if not useless.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ADMINISTRATIVE CONTROL OF UNDERGROUND WATER: PHYSICAL AND LEGAL ASPECTS

Discussion

BY L. STANDISH HALL, ASSOC. M. AM. SOC. C. E.

L. STANDISH HALL,[46] ASSOC. M. AM. SOC. C. E. (by letter).[46a]—A thorough treatment of the physical and legal aspects of the control of underground water is presented in this paper. The many and varied possibilities with reference to the physical conditions are adequately expressed in the sentence (see heading, "Underground Water in General"), "each situation is a separate study which must be exhaustive and widespread to reach reliable conclusions". The only generality that can be expressed is that all surface, vadose, and ground-waters were derived primarily from rainfall. (Vadose waters[47] are the suspended sub-surface waters found in the zone of aeration between the land surface and the water-table.)

The author states that ground-water cannot be developed without reducing the flow of the adjacent streams. This statement is true, in general, because it was implied that the streams in question were perennial, and that the development of surface-water diversions was possible. Such streams in semi-arid regions usually rest on the water-table and, to some extent, are effluent, or they may be both influent and effluent along various stretches of the channel. A ground-water development in these areas will naturally result in a lowering of the water-table, but if the pumping is not excessive a balance will be reached ultimately subject to the normal fluctuaexcessive a balance will be reached ultimately subject to the normal fluctuation of ground-water levels resulting from the natural variation in rainfall.

NOTE.—The paper by Harold Conkling, M. Am. Soc. C. E., was published in April, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1936, by Messrs. Joseph Jacobs, and W. D. Faucette and J. E. Willoughby; September, 1936, by R. E. Savage, Assoc. M. Am. Soc. C. E.; December, 1936, by Messrs. H. J. F. Gourley and O. J. Baldwin; January, 1937, by Messrs. G. E. P. Smith, and David G. Thompson; February, 1937, by Wells A. Hutchins, Esq; and March, 1937, by Messrs. John E. Field, George S. Knapp, and Chandler Davis.

[46] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

[46a] Received by the Secretary February 27, 1937.

[47] "Outline of Ground Water Hydrology", by Oscar E. Mainzer, *Water Supply Paper No. 494*, U. S. Geological Survey, p. 22.

This balance is reached from the increased seepage or percolation from the stream induced by the steeper gradient from the water level in the channel to that in the adjacent ground-water body. If the streams were originally influent, a change to effluent will result.

A natural limit to the process of increasing percolation from ground-water development is reached if the ground-water is lowered below the stream bed at points adjacent to the channel. When this condition occurs, no further lowering of the water-table will increase percolation under a given flow. The stream then would have reached the natural condition existing in ground-water adjacent to intermittent and ephemeral streams, where stream channels are above the water-table at all times. Under such conditions, percolation depends on the volume and duration of flow, and ground-water withdrawals exert no influence on the recharge.

If surface-water irrigation has developed in a basin prior to the use of the underground water, it does not appear possible, on a perennial stream, to permit a subsequent pumping development under administrative control. If the surface-water use occurs on an intermittent or an ephemeral stream, the ground-water development would not conflict with the surface-water use. However, if surface-water use on the latter type of streams succeeded ground-water development, there would be a conflict of rights, since diversion of surface flow would reduce the quantity that would percolate.

Even a subsequent ground-water development on a perennial stream subject to appropriation, must occur up stream from the point of diversion to be of damage since (as is noted by the author) excess water is always carried from the stream with surface irrigation and frequently results in a rising water-table in the lower part of the irrigated area. Ground-water development down stream from the surface irrigation would frequently be a benefit and, in fact, is sometimes resorted to under a canal system to prevent water-logging of the lower areas.

The economics of the situation may also enter into the extent that development of ground-waters may progress unhindered. Sufficient lowering of the water-table eventually would result in excessive costs so that some users would be forced to abandon their development. Here, again, priority might establish the justice of such a method of survival depending on whether or not the lands forced out of cultivation were first in the ground-water field but, because of their location, were the first to feel the effects of mounting water costs.

The variety of conflicts that may arise in connection with ground-water uses and surface-water diversions would lead to the careful consideration of the author's conclusion (Item 19) that where fullest development of water resources are desired, "administrative control of underground water should not be adopted until over-draft threatens, and then only in particular areas." Furthermore, "a step at a time may be the safest course."

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DISCUSSIONS

ANALYSIS OF CONTINUOUS FRAMES BY BALANCING ANGLE CHANGES

Discussion

BY MESSRS. FRED J. BENSON, AND DAVID B. HALL

FRED J. BENSON,[30] JUN. AM. SOC. C. E. (by letter).[30a]—The method of analyzing continuous frames by balancing angle changes, presented by Professor Grinter, is a valuable alternative to the method of balancing fixed-end moments. Professor Grinter discusses briefly the use of the device of successive distortions in applying his method to structures having more than one story or one panel. This important application seems to deserve more attention. The following analysis will serve to illustrate and clarify the points emphasized in the paper.

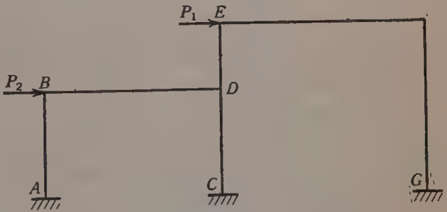

FIG. 20.—TWO-STORY BENT WITH LATERAL LOADS

The two-story bent shown in Fig. 20 has two degrees of freedom of horizontal movement. Hence, two successive distortions must be produced in order to determine the moments produced by the lateral loads. In order to produce the first distortion it is assumed that Joints E, F, and G are fixed in position and that the remaining joints are pin-connected. An unresisted horizontal movement of Joints B and D is produced in the direction of the load, P_2. This movement produces equal angle changes at the top and bottom of each column affected. Such angle changes are then balanced by Professor Grinter's method. The shear in each column is computed from the balanced angle changes and the produced horizontal movements by Equation (5). The total shear in each story is determined by the summation

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NOTE.—The paper by L. E. Grinter, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on this paper has been published in *Proceedings*, as follows: December, 1936, by Messrs. John E. Goldberg, G. A. Maney, Paul Andersen, and William F. Luce; and March, 1937, by Messrs. L. J. Mensch, Marvin A. Gray, and A. Floris.

[30] Instr., Testing Materials, Civ. Eng. Dept., Purdue Univ., West Lafayette, Ind.

[30a] Received by the Secretary March 2, 1937.

of the column shears. These shears may be designated as V_{1a} and V_{2a}. In order to produce the second distortion it is assumed that Joints A, B, and C are fixed in position and that the remaining joints are pin-connected. An unresisted horizontal movement of Joints E and F is produced in the direction of the load, P_1. Again, the angle changes produced by this movement are balanced and the story shears, V_{1b} and V_{2b}, are obtained.

The two distortions must be combined in such a manner that the true story shears are obtained. Therefore, it is necessary to determine the factor by which the balanced angle changes and produced horizontal movements for each distortion must be multiplied to obtain the true story shears. If x represents the factor for the first distortion, and y, the factor for the second distortion, the following equations may be written:

$$x\,V_{1a} + y\,V_{1b} = P_1 + P_2 \quad \text{.....................} \quad (41\,a)$$

and,

$$x\,V_{2a} + y\,V_{2b} = P_1 \quad \text{..........................} \quad (41\,b)$$

Equations (41) are solved simultaneously for x and y. The true rotation of any joint is found by multiplying the balanced angle change for each distortion by the corresponding factor and then adding these two corrected angle changes. The true horizontal movement of any joint is found in the same manner by applying the factors to the produced horizontal movements. The movement in any member at any joint is then found by using the slope-deflection equations. Since the summation of moments at any joint must equal zero, it is quite evident that the method is self-checking.

The rapid use of either the methods of balancing fixed-end moments or angle changes requires considerable practice. In general, the final moments can be obtained more rapidly by balancing fixed-end moments. However, for structures containing a number of joints, the fact that the method of balancing angle changes is self-checking is of considerable importance since it offers a rapid method of checking the solution without repeating the balancing process.

David B. Hall,[31] Assoc. M. Am. Soc. C. E. (by letter).[31a]—In the section of this paper entitled "Haunched Beams and Curved Members," the author calls attention to the fact that end-angle changes, and other factors, must be specially calculated for such members. The following observations may assist in the calculation of the angle changes due to loads.

A haunched beam with a unit moment at the left end is shown in Fig. 4 (Point A), and below it is shown the deflection curve. This diagram is the influence line for the angle change at Point A due to loads anywhere on the span, because, by the theorem of reciprocal deflections, the deflection at any point due to a unit moment at the end is identical to the rotation at the end due to a unit load at that point.

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[31] Asst. Engr., State Dept. of Public Works, Albany, N. Y.

[31a] Received by the Secretary March 6, 1937.

If the method of elastic weights is used to compute this curve, it will be found expedient to use a double summation process, adding up elastic weights to obtain shears between successive points, and adding up these shears, times length of divisions, to obtain moments. If this plan is regarded as a subterfuge, the identical operations may be performed, treating the elastic weights as angular changes (which they are); adding them to obtain slopes, beginning with ϕ_A as the slope at the end; and summing up the products of slopes and division lengths, to obtain the deflections.

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DISCUSSIONS

SELECTION OF MATERIALS FOR ROLLED-FILL EARTH DAMS

Discussion

BY R. R. PROCTOR, M. AM. SOC. C. E.

R. R. PROCTOR,[34] M. AM. SOC. C. E. (by letter).[34a]—Five important requirements for soils that are to be used for the construction of rolled-fill earth dams are recorded in this paper. The requirements for water-tightness and stability are of the utmost importance; the dam may fail if the soils are not sufficiently compacted to resist excessive softening or percolation that may cause erosion or piping. Mr. Lee has shown that the grading of a soil determines the minimum porosity that can be secured by use of a particular method of compaction.

The writer does not make use of grading curves, as described by Mr. Lee, because his experience has indicated that laboratory and field experiments require less time and expense, and that such results are more readily adapted to the control of construction methods.[35] This is an absolutely essential requirement for any method of soil analysis or classification; no amount of detailed laboratory investigation of soils is of much value unless there is certainty that the soils will be compacted in the dam as the engineer requires.

Until a better method is developed, the writer prefers to use the plasticity needle penetration resistance when the compacted soil is fully saturated, for a guide to stability. The results of percolation tests, first made at the minimum soil density required for stability, are used as a guide for safety from excessive percolation. It has been found that a given method of compaction may produce a soil density of 12 lb per cu ft more than that required for a saturated penetration resistance of 300 lb per sq in., whereas, in other

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NOTE.—The paper by Charles H. Lee, M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1936, by Messrs. T. T. Knappen, and Paul Baumann; January, 1937, by Messrs. William C. Hill, A. Floris, and Fred D. Pyle; February, 1937, by Messrs. Joel B. Cox, Stanley M. Dore, John E. Field, William P. Creager, and Joseph Jacobs; and March, 1937, by Messrs. C. H. Kadie, Jr., and Ralph Bennett.

[34] Field Engr., Bureau of Water Works and Supply, City of Los Angeles, Los Angeles, Calif.

[34a] Received by the Secretary February 23, 1937.

[35] *Engineering News-Record*, August 31, September 7, 21, and 28, 1933.

soils, such a saturated penetration resistance cannot be secured by this method of compaction. No other explanation than variation in grading has been found; however, the writer feels that the suitability of such soils is better established by actual tests.

An illustration of this fact is seen in the design of a large earth-fill dam proposed by the writer. The proposed borrow-pit was once selected for a rock quarry for a rock-fill dam. Without going into geologic details, subsequently revealed by studies of thin sections of the apparent rock under a petrographic microscope, the apparently hard and durable rock was a metamorphosed dioritic gneiss, so badly fractured that it readily broke down into an apparent soil as it was excavated and loaded into trucks. The writer had the usual compaction-plasticity curves prepared, which indicated that successful compaction of this material was possible. Several small compacted test fills were then made, using progressively heavier rollers until a roller weight and number of roller trips adequate to accomplish a soil density sufficient for a saturated penetration resistance of 300 lb per sq in. were determined.

A typical compaction curve for this material is shown in Fig. 22. Of this specimen, 43.8% was retained on a No. 4 sieve and 23% passed a No. 200 sieve. The specific gravity was 2.79. The laboratory compaction curve was prepared from the 57.2% that passed through the No. 4 screen. The extent of the field compaction was determined by measuring the density of the compacted material in place, and then calculating the density of the fines by deducting the weight and volume of all the material retained on the No. 4 screen. It will be noted that the field compaction was only 1 lb per cu ft in excess of the required minimum. This was caused by the 43.8% of rock (material retained on the No. 4 screen); other samples of similar material, but containing less rock, gave results comparable to the laboratory compaction curve. It will be noted that a compacted penetration resistance

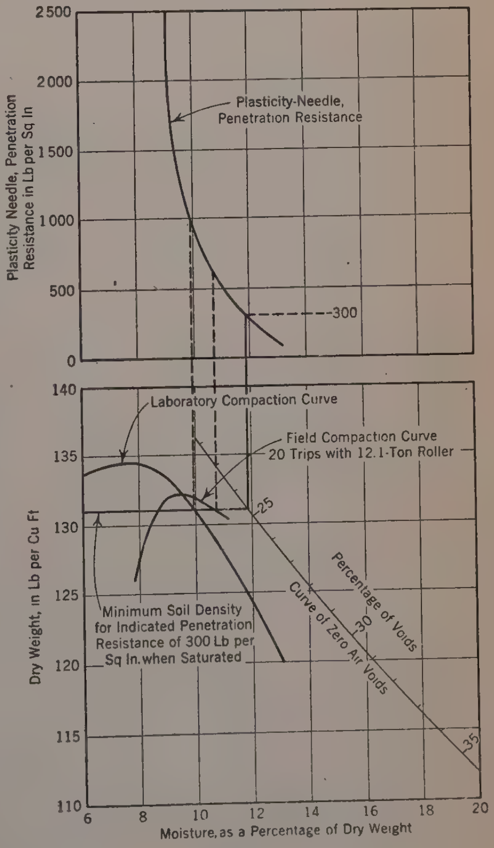

FIG. 22.

of 600 lb per sq in. represents the maximum allowable moisture content for compaction of this material; the addition of more water limits the compaction to less than the established minimum. It will also be noted that the soil can be compacted at penetration resistances greater than 2 500 lb per sq in. It is desirable to compact this soil at a compacted penetration resistance range from 2 000 to 3 000 lb per sq in., in order to reduce the cost of truck haul over the fill and to provide for consolidation during the construction period without removal of water. thus eliminating most of the slow settlement that often occurs after a dam is finished.

It is interesting to note the effect of compacting this material to a lower dry weight as, for example, that of 120 lb per cu ft. The laboratory compacted sample at this dry weight had a penetration resistance of 100 lb per sq in. as compacted, which, in the writer's opinion, is too low a value for safety in a large dam. The moisture content of this soil could increase, without change in dry weight, from 13.1% to 16.3% of the dry weight, upon saturation. The plasticity of this saturated material may be compared to that of a concrete mix of 1:2.46:2.95, dry rodded volume, with 7.1 gal of water per sack of cement. This is obviously too soft for use in a dam. This discussion clearly indicates the need for more than soil-grading curves to determine the suitability of some soils for use in earth dams.

The writer has had sufficient experience with various types of soils to appreciate the fact that the methods proposed by Mr. Lee are adequate to determine the suitability of most soils, but actual tests are needed for doubtful types. Tests are also required to determine the moisture content before compaction, the required compacted dry weight of soil, and to check the results of construction operations, regardless of the degree of suitability the soils may possess. This work can be started a short time before actual construction begins, if lack of funds or other causes prohibit much preliminary experimental work.

Mr. Lee's paper required considerable effort and time to prepare, and a continuation of these efforts should be made to develop more data of this nature.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

INTERACTION BETWEEN RIB AND SUPERSTRUCTURE IN CONCRETE ARCH BRIDGES

Discussion

By MESSRS FRANK M. BARON, A. A. EREMIN, AND HARDY CROSS

FRANK M. BARON.[26] ESQ. (by letter).[26a]—A contribution to engineering literature which attempts to present and use a designer's viewpoint is to be highly commended. Many readers may fail to grasp the intrinsic value of Mr. Newmark's paper. The method of attack, as applied to the problem at hand, is certainly as valuable as his conclusions. It is an approach of this kind that yields fruitful information to an engineer as to the probable action of a structure. Too much of the literature recently is being deluged with methods of analysis, and sight is being lost of every engineer's goal—that of a successful design. The interpretation of analyses and tests, and the significance of factors governing a design are usually more important than the means of obtaining the required data.

A designer usually wants two types of answers to a problem—qualitative and quantitative. A qualitative study of the probable action of a structure will generally lead to the quickest estimate of the quantities required. A quantitative study gives only a quantitative answer, and does not lead into possibilities of structural control, unless one is to carry it to limits of comparative analysis. Analysis of any kind involves some assumptions, and, as a result, judgment must be trained as to the interpretation of the resulting data. One of the greatest follies in thinking is the acceptance of analytical results without properly weighing the assumptions and the probable action of the structure involved. This is where the true worth of the author's paper is distinguished. The method of attack is applicable not only to arches, but also to any type of continuous frame.

In relating the action of an arch in terms of the classification of indeterminate structures presented in 1935 by Hardy Cross, M. Am. Soc. C. E.[2],

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NOTE.—The paper by Nathan M. Newmark, Jun. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: February, 1937, by A. Floris, Esq.

[26] Asst. Research Engr., Chicago Bridge & Iron Co., Chicago, Ill.

[26a] Received by the Secretary, February 8, 1937.

[2] "The Relation of Analysis to Structural Design," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1363.

the author has presented his viewpoints on arch action analogically. Exhaustive research was involved, as one can see that the necessary data must have been gathered through the analysis and compilation of the experiences of various engineers before any interpretation of results was possible.

In considering the significance of over-stress, the author makes a distinction between stresses depended on to "carry the load" and participation stresses. It stands to reason that participation and deformation stresses should be discounted if they do not impair a member from carrying the load, or are of localized nature. In several cases the author seems to have attached too much importance to over-stress due to participation action. Such over-stress does not necessarily mean that a structure will fail. Consider the case of over-stress in the column sections, for example. What if the over-stress due to interaction is so great that the column section should crack? The member may crack, but then the structure would not fall, as the columns would still perform their primary purpose of supporting the deck, and transmitting the deck loads to the rib. If cracking is considered as a failure, then some means of articulation may be adopted at times to prevent it.

It is true that, if the column is relied upon to help the rib, and the column should crack, it might prove disastrous to the rib; but this raises the question as to whether one would try to reduce the rib section in an attempt to obtain a more economical design due to help from participation stress. A few figures can be attached to the relative costs of the elements of the structure to show the futility of this choice[27]. In some cases, the deck is probably the only element that might justify consideration of interaction help from the columns or from the rib.

As to the matter of change in the sign of the moment at the end of the panel due to interaction effect, the author has lost sight of a point that needs further emphasis. It is the importance of dead load and live load ratio upon the consideration of this effect. Then, also, is this the stress that governs the depth or cross-section of the deck? It may be important at times only from consideration of the proper placing of steel.

The selection of the type of arch and the importance of interaction effect must also be made from the standpoint of dead load and live load ratio. A long span arch is usually a dead load structure, and may be shaped so that the pressure line follows the arch axis for dead load. Interaction would not be important in this case, as the ratio of dead load to live load is usually high. In the case of a short-span, open-spandrel arch, live load may predominate, and participation becomes a problem. It is preferable to have a high dead load ratio, and the selection of a filled spandrel arch might be the answer, provided other factors remain equal. In the medium span arch then, apparently, is where the problem of interaction must usually be decided.

The flitch-beam concept is outstanding in the possibilities that it affords a designer. Through its use, the engineer is able to predict the degrees of stress, but what is even more important is its adaptability to structural control. The effect of one member upon another may be studied, and a choice made as to the final structure desired. Instead of having the moment curve virtually "wag" the structure, proper selection of $\frac{\text{depth}}{\text{length}}$ ratios enables the designer to "wag" the moment curve. Structures try to do what the designer wants them to do as much as they can. Judgment, therefore, is required as to whether the structure can act as "instructed."

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[27] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., p. 306.

The writer wishes to raise the question as to what a consideration of interaction effect does to the safety factor of the entire structure. The author hints at it as applied to a column section. The importance of interaction in the members of an arch may often be determined through considerations of comparative safety factors. If interaction effect is depended upon to help the rib, then a much lower safety factor would result, and probably forbid the use of a smaller rib section. In considering participation stress in the column, the writer believes the difference in total safety factor would be so slight that a designer may easily neglect interaction. The difference involved in the deck would depend entirely upon the ratio of dead load to live load, as well as the judgment upon the discounting of participation stresses.

Concluding Remarks.—The paper is one of the few in which the arch is considered as a whole structure. Judging by the time spent in the analysis and design of the rib, many designers seem inclined to assume that this is the only part of an arch, whereas the rib usually has the smallest effect on the total cost of the structure. The paper is further enlightening upon the need of interpretation as applied to results and data obtained in the field of structures.

A. A. EREMIN,[28] ASSOC. M. AM. SOC. C. E. (by letter).[28a]—An interesting study of the interaction stresses in the rib and superstructure of concrete arch bridges is presented in this paper. However, for conclusive results the arch bridges shown in Fig. 1 seem to have spans that are too short. The theoretical and experimental study should be extended to arch bridges with larger spans and a greater number of panels. In practice, it is seldom that an open-spandrel arch bridge is constructed with less than ten spandrel panels.

In Item 5 of "Summary of Results of Analyses", the author refers to temperature and deformation strains instead of stresses as it seems he should have done. Apparently, temperature stresses in the deck may be of the same order as those in the arch only if the relative stiffness of rib, spandrel colums, and the spandrel girders in the arch bridge are approximately the same and the bridge can be considered as a Vierendeel arched truss.

The author's statement (see heading "Treatment of the Arch with Superstructure as a Flitch Beam") that "the stresses due to interaction flexure in the girder is to the flexural stress in the rib approximately as the depth of the girder is to the depth of the rib", needs clearer interpretation. Theoretical and experimental studies available at present are not sufficient to support it. It seems evident that the relative stiffness of the straight spandrel girders and the arched rib cannot be compared in terms of a depth ratio only, without reference to the ratio of the arch rib rise to the span length. For complete study of such interaction the analysis of stresses and a comprehensive inspection of existing arch bridges may produce much useful information.

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[28] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[28a] Received by the Secretary January 30, 1937.

The interaction stresses in open-spandrel arch bridges vary with: (1) The ratio of the live load to the dead load considered in the bridge; (2) the ratio of the arch rib rise to its span length; (3) the height of the spandrel columns at the crown of the arch rib; and (4) the degree of restraint imposed to avoid variations in the span of the arch.

In long-span arch bridges the live load stresses are generally small as compared with dead load stresses. Therefore, the former have a negligible effect on displacements in the spandrel girders and arch ribs. In the Plougastel (arch) Bridge, in France[29], with a span length of 611.39 ft, a rigid double-deck, reinforced concrete spandrel truss was used. However, it had only two expansion joints, 116.28 ft from the center of the span. Evidently, the effect of the live load and temperature deformations in the arch rib on the stresses in the spandrel trusses was negligible.

In arch bridges with shorter spans the ratio of live load stresses to dead load stresses in ribs is larger. Therefore, the effect on stresses in the superstructure, of displacements in the rib produced by live load and temperature stresses is greater. Various devices have been used to reduce the participating stresses in superstructures. In the Broakpark (arch) Viaduct, at Cleveland, Ohio,[30] with a span length of 192 ft 3 in. from center to center of piers, a flexible roadway deck slab served this purpose, whereas in the Wandau Enns (arch) Bridge, in Germany,[31] with a span length of 242.06 ft, expansion joints were provided at each spandrel column.

As the span length is further decreased more difficulties are encountered in making the superstructure sufficiently flexible to accommodate a possible displacement in the rib. An interesting case of a flexible roadway deck slab, with overhanging ends cut by expansion joints, is that of the arch bridge near Donner Lake, in California.[30] This structure has a span of 110 ft, divided into eleven spandrel panels. The roadway deck is on a grade of 7% and the center line of the roadway is curved to a radius of 360 ft. Nevertheless, the slab is in excellent condition and reveals no visible cracks.

In concrete arch bridges of short span lengths, and with heavy railroad loading, the ratio of live load stresses to dead load stresses in the ribs is large. Therefore, it is practically impossible to make the deck slab sufficiently flexible. In such structures, rigidity is generally governed, not by the ribs but by the spandrel girders and spandrel columns. Ribs are made flexible enough to reduce the participating stresses in the connecting spandrel columns. A fine illustration of this type of bridge is the Voltschiel (arch) Bridge, near Andeer, in Switzerland,[32] in which the rigidity of the spandrel columns was increased by constructing massive abutment piers founded on rock. It is more difficult to provide sufficient rigidity in the spandrel columns in arch bridges with high abutment piers. Generally, in this case, the spandrel girders are rigidly connected to arch ribs at the crown similar to that used in the Landquart (arch) Bridge, at Klosters, Switzerland.[33] Of course, the tractive forces in this case are thrown in the arch and produce unsymmetrical stresses in the ribs. Lateral forces should be included among the principal sources of stress indicated by the author. The major lateral forces in arch bridges are generally produced by wind pressures which are negligible only in arch bridges with short spans. They cannot safely be neglected in arch bridges with large spans. The difficulty of analyzing interaction stresses when the spandrel girders are rigid, when the columns must resist combined forces in the plane of the rib, and when the structure is subjected to lateral forces, is a serious one. The stresses in the ribs can be determined more readily if the spandrel girders are broken by expansion joints. The lateral forces from the spandrel girders are then transmitted immediately to the arch ribs and the lateral stresses in the ribs can be computed by a simple method presented by the writer elsewhere.[34] In arch bridges with continuous, rigid, spandrel girders and columns, the lateral stresses are carried to the high abutment piers and this design could scarcely be considered as the most economical one.

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[29] *Le Genie Civil*, October, 1930, p. 317.

[30] *Engineering News-Record*, October 11, 1934, p. 467.

[31] *Beton und Eisen*, March 5, 1931, p. 73.

The following tentative conclusions may be made in the light of this discussion:

(1) Open-spandrel arch bridges with spans greater than 120 ft, should be designed with a flexible roadway deck slab, or the spandrel girders should have expansion joints.

(2) Rigid spandrel girders and columns may prove the most economical for arch bridges with spans less than 120 ft. (If the structure is to be designed for railroad loading the span may be increased to 200 ft); and,

(3) If it is anticipated that foundations may settle or that elastic piers may be otherwise displaced enough to change the span length, spandrel girders and the arch rib should each have approximately the same degree of rigidity, for the most economic design. In such a case the structure will be the same as that generally called the Vierendeel arched truss.

Hardy Cross,[35] M. Am. Soc. C. E. (by letter).[35a]—This paper is worth study by those interested in interaction of rib and deck, a phase of arch design that in recent years has attracted some attention, especially in connection with the work of the Special Committee on Concrete and Reinforced Concrete Arches.[36] The paper raises the question as to whether analysis which includes the effect of the deck structure is really worth while. The Committee recommended that such analyses be made, but failed to indicate in any way what was to be done with them. Whether they should be made certainly depends largely on the relative importance of live load, dead load, and temperature distortions as affecting the total stress. Usually the ribbed arch of concrete is chiefly a dead load structure; the live load effect has become relatively important only with modern loading. Interaction is chiefly important in short spans because live load stresses may be relatively greater and also because there is more pronounced interaction. Dead load stresses are primarily compressive; live load stresses are chiefly flexural. If the live load stress becomes important it is much more useful to ask how this flexure should be provided for than merely to analyze a design more or less arbitrarily chosen.

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[32] *Engineering News-Record,* March 4, 1926, p. 357.

[33] *Loc. cit.,* January 11, 1934, p. 44.

[34] *Proceedings,* Am. Soc. C. E., December, 1935, p. 1453.

[35] Prof. of Structural Eng., Univ. of Illinois, Urbana, Ill.

[35a] Received by the Secretary March 4, 1937.

[36] *Transactions,* Am. Soc. C. E., Vol. 100 (1935), p. 1427.

Assuming that an analysis is to be made, the question is, then, whether it will furnish a satisfactory guide as to what should be done to improve the design. Thus, if over-stress is indicated in the columns, the question at once arises whether it would be wiser to make the columns larger, or to make them smaller; either answer may be satisfactory in some cases. Similarly, over-stress in the deck may indicate either that the deck should be made stronger or possibly that it should be made more shallow. In general, any hope of real improvement of design will depend a great deal on whether or not the designer can foresee clearly some simple way in which a structure is to carry its load; complicated analyses, unless interpreted very judiciously, rarely lead to simple pictures of structural action. This paper does furnish some simple pictures helpful to the designer.

The paper has value in drawing attention to types of open-spandrel arches designed to resist flexure in other ways than in the rib alone. It is possible to carry all flexure in the deck, that in the rib producing participation stress; it is possible (and may sometimes be desirable) to carry the flexure partly in the deck and partly in the rib. The paper makes clear that any attempt to divide the flexure between rib and deck will depend for its success upon the relative depths. Structures designed for flexural resistance in the deck have been used on the continent of Europe; they are worth some study.

It may be worth while to analyze such structures in the hope of predicting the type of interaction that can be secured, assuming that such interaction is desirable, or it may be worth while to analyze to determine whether, for a more or less arbitrarily assumed layout, such interaction is dangerous. There does not seem to be now any satisfactory way of telling from the analysis whether the condition is really dangerous. Mere computations of stress alone do not indicate the degree of safety any more than they do when secondary stresses are included in analyzing steel trusses. Thus, as Mr. Newmark points out, over-stress in a column due to participating flexure does not very seriously threaten the safety of the column and does not threaten at all the safety of the arched structure as a whole.

The writer is gratified to see a paper which diverts attention from algebraic manipulation to a consideration of the objectives of such manipulation.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

STRUCTURAL APPLICATION OF STEEL AND LIGHT WEIGHT ALLOYS

A SYMPOSIUM

Discussion

By RALPH FREEMAN, M. AM. SOC. C. E.

RALPH FREEMAN,[179] M. AM. SOC. C. E. (by letter).[179a]—The evolution of high strength steels in structural engineering in Great Britain has led to the adoption of qualities of steel differing materially from those used in the United States, as recorded in the paper by Mr. Moisseiff. Nickel steel has not been used to any significant extent for bridges or for other structures designed in Great Britain, no doubt because no bridge has been constructed having a span long enough to require the use of a superior quality of steel, obtainable only at a material increase of cost.

As stated by Mr. Moisseiff (see heading, "History of Silicon Steel"), "silicon" steel was first produced in 1907 for the construction of part of the upper structure of the *Mauretania* and data regarding this steel and the silicon steel in the *Lusitania* have been reported by Sir Robert Hadfield.[180]

The plates rolled were of large dimensions (as long as 38 ft and $1\frac{1}{4}$ in. thick, the width varying from 7.0 to 8.1 ft), requiring ingots as heavy as 16 tons. The plates were also subjected to the most severe tests in order to prove their resistance to sudden shock and violent stress beyond the yield point. In one series of these tests plates, 6 ft by 3 ft by 1 in. thick, were placed on supports 4 ft apart and tested by dropping a 3-ton steel

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NOTE.—This Symposium was presented at the meeting of the Structural Division at Pittsburgh, Pa., October 14–15, 1936, and published in October, 1936, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: December, 1936, by Messrs. E. Mirabelli, R. W. Vose, Raymond H. Hobrock, William F. Clapp, J. C. Hunsaker, Horace C. Knerr, and F. T. Sisco; January, 1937, by Messrs. J. Charles Rathbun and D. M. MacAlpine, Fred L. Plummer, C. F. Goodrich, G. K. Herzog, John H. Meursinge, P. C. Lang, Jr., and W. L. Warner; February, 1937, by Messrs. Elmer K. Timby, Werner Lehman, Otis E. Hovey, and R. G. Sturm; and March, 1937, by Messrs. E. Robert deLuccia, O. J. Horger, A. W. Demmler, Theodore Belzner, J. P. Growdon, Karl Arnstein, A. Christianson, Robert E. Glover, Arthur C. Ruge, Alexander Klemin, Paul E. Gisiger, Russell C. Brinker, Arshag G. Solakian, and H. D. Hussey.

[179] Managing Partner (Sir Douglas Fox & Partners) Westminster, London, S. W. 1, England.

[179a] Received by the Secretary March 13, 1937.

[180] "Metallurgy and Its Influence on Modern Progress", by Sir Robert Hadfield.

ball on them from heights increased successively to 31 ft. Presumably, this was the maximum practicable value since the plates did not crack although they buckled.

In spite of the apparent successful application of this steel for shipbuilding, it does not appear to have been used again for this purpose and was not adopted for bridge building until the construction of the Sydney Harbor Bridge during the period, 1923–1928.

When preparing the designs for the Sydney Harbor Bridge, the writer considered the advisability of using various alternative qualities of steel, from mild steel (with a yield stress of about 15 tons per sq in.) to alloys having a yield point of 50 tons per sq in. The conclusion was reached that no high-class alloy steel effected a saving in weight which justified the extra expense but that "silicon" steel, having a yield stress of 20 tons per sq in., or one-third higher than that of mild steel, would effect a substantial reduction of cost. This steel complied with the following analysis:

	Percentage
Carbon	0.32 to 0.42
Manganese	0.60 to 1.00
Silicon	0.15 to 0.25

The rolled material included plates as wide as 8.25 ft and 2.25 in. thick, weighing 10 tons, and angles of 12 in. by 12 in. by 1.25 in. The permitted working stresses were 30% higher in tension and 25% higher in compression. The cost of the raw material was about 25% in excess of that of mild steel. No special difficulties of fabrication were experienced and the cost of fabrication was little more than that of mild steel. Nearly 40 000 tons of this steel were used for the Sydney Harbor Bridge, but as far as the writer is aware, "silicon" steel has not been used by British engineers for any other structural work.

This circumstance, no doubt, arose from the fact that at the time of the construction of Sydney Harbor Bridge, attention was being given in Great Britain to the production of high tensile steel superior to "silicon" steel, especially for shipbuilding, and led to the production of various alloys containing small percentages of manganese and chromium which could be produced at a cost very little more than that of silicon steel and would have distinctly superior physical properties.

These high-strength tensile steels are now extensively used for shipbuilding, bridge construction, and other structural steelwork. Steel used for the latter purpose is defined by a British Standard Specification which declares the physical properties of the steel, but does not state the chemical analysis. The yield point must be higher than 23 tons per sq in. for sections or plates not exceeding 1.25 in. in thickness, reduced to 22 tons for thicknesses up to 1.75 in.

Actual manufactured plates and sections of the kind in most general use (that is, of thicknesses as great as 1 in.), have a yield stress of 24 tons per sq in. The breaking strength is specified to be between 37 and 43 tons per sq in. The quantity of steel of this character produced in recent years in Great Britain exceeds 300 000 tons.

The steel of this quality which has been produced most extensively is a chromium-copper-manganese alloy complying with the following composition:

	Percentage
Carbon (maximum)	0.30
Chromium	0.70 to 1.0
Copper	0.25 to 0.50
Manganese	0.70 to 1.0
Silicon	0.20

Another alloy contains 1.0% to 1.5% of manganese, as much as 0.5% of copper, and no chromium. Other types contain smaller proportions of chromium and copper.

High tensile steel plates have a tensile strength transverse to the direction of rolling equal to that in the direction of rolling and sustain the usual cold bend tests (bending through 180° over a diameter equal to three times the thickness of the test piece without cracking).

The yield stress of high tensile steel of the foregoing type is approximately 50% higher than that of British mild steel. The permissible stresses allowed for this quality of steel are usually taken as 50% in excess of those allowed for mild steel.

The raw material cost of British "high tensile" steel can be taken roughly as 25% in excess of that for mild steel, and fabrication costs are not significantly higher, except as the consequence of the reduced tonnage resulting from the use of a stronger material.

Rivet steel commonly used for structural steelwork has a tensile stress of 26 to 30 tons per sq in., and it has been felt that with rivets of this class full advantage could not be taken of the superior strength of high tensile steel. Rivet steels of improved quality are produced, having tensile strengths (before driving) as high as 30 tons per sq in., and a strength in shear in high tensile plates exceeding that of mild steel rivets in mild steel plates by about 50 per cent. These rivets have not yet been used extensively.

Modern "evolution" of high strength steels when all is said appears to be anything but evolution when one achievement is recalled which occurred in an age now apparently forgotten in its own country. The steel used for the arch ribs of the Eads Bridge over the Mississippi River, at St. Louis, Mo. (completed in 1874), is not only one of the most astonishing features of this wonderful bridge, but an example of quality in a structural material that has probably never been surpassed.

In 1881, Mr. C. N. Woodward recorded[181] the fact that the steel staves that formed the ribs of the arches, were of chromium alloy which rolled very well. This was in 1872. The steel actually used is said to have had a tensile strength of 120 000 lb per sq in. Tests made in 1869 for the late James B. Eads, F. Am. Soc. C. E., apparently of the steel proposed for the bridge, indicated a superior quality. One series of these tests, made at Messrs. Kirkaldy's Testing House, in London, in 1869, of chrome steel bars showed that the limit of elasticity in tension had an average value of 82 000 lb per sq in., the corresponding average ultimate strength being 155 000 lb per sq in.!

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[181] "A History of the St. Louis Bridge", by C. N. Woodward, pub. by G. I. Jones & Co., 1881.

The writer subscribes entirely to Mr. Moisseiff's views regarding the advantages and limitations of high strength steel. He has found that high tensile steel is a source of great economy for long spans and has the effect of greatly increasing the economic span. It is also economical for small spans where freight costs are high. For a light road bridge recently built in Rhodesia, crossing a river, 1 000 ft wide, with difficult foundation conditions, it was found to be advantageous to use a single arch span of high tensile steel with no river piers. The use of such steel made this type of bridge practicable. Had mild steel been used the bridge would have been of a different type (girder spans on piers) and would have been far more costly.

Another bridge carrying a precisely similar roadway and having the same span is now (1937) about to be built, but in a locality where the freight cost will be about £16 per ton. In this case, the suspension type of bridge with high-tensile steel stiffening trusses is the least costly the economy arising to a large extent in consequence of its smaller total weight and the reduction of freight costs.

The saving of freight costs to distant parts of the British Empire, from Great Britain, by using high tensile steel, is also in many cases the means of affecting considerable savings. For sites where the total freight may be as much as £16 per ton, this will be readily appreciated.

In using high tensile steel for bridge structures the writer has paid particular attention to reducing to a minimum the number of different sections to be used, and finds that when this aspect of design is carefully studied the small number of sections which is really essential is somewhat surprising. With a class of steel which is not in general use this is important; otherwise, costs may be greatly increased.

The possible combination of normal and high tensile steels is frequently advantageous and is made readily practicable by the fortunate, but somewhat remarkable, fact that for all steels used for structural purposes the modulus of elasticity is the same. As far as the writer has been able to ascertain, there is no more variation between values for the modulus of elasticity for different kinds of steel than there is for different specimens of the same kind. This uniformity of the modulus permits the interconnection of various classes of steel without risk of indeterminate stresses which would be caused by different moduli of elasticity.

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DISCUSSIONS

STRESSES AROUND CIRCULAR HOLES IN DAMS AND BUTTRESSES

Discussion

By Messrs. J. H. A. Brahtz, V. L. Fedorov, and F. W. Hanna

J. H. A. Brahtz,[16] Esq. (by letter).[16a]—A new approach is presented in this paper, in the analysis of the stresses in the vicinity of a cylindrical opening in a triangular gravity dam when the axis of the opening is parallel to the face of the dam. The main assumptions are that: (2) The water surface is at the theoretical apex of the dam; (2) the opening is far removed from the faces of the dam; (3) the material is perfectly elastic; and (4) the tensile stresses are so small that the concrete does not crack and, if it does crack, that sufficient steel reinforcement will make the dam act as a homogeneous elastic material.

The method is also applicable to a triangular buttress of a hollow dam if the elastic interaction between the up-stream slab or arches and the buttress is ignored. Even if the results obtained are not exact (and, in one instance, incorrect in principle, as will be shown subsequently), it is believed that stresses computed by the author's method will agree quite closely with those obtained by the approximate methods used by the U. S. Bureau of Reclamation. One of these methods, also based on Assumptions (2) to (4), uses the exact solution by Kirsch for a circular hole in an infinite plate acted upon by uniform boundary forces equal to the principal stresses computed in the structure at the center of the opening as if the opening did not exist.[17]. This method is far simpler in application than that developed by Mr. Silverman and is useful not only for the specific type of structure and loading treated by him, but for any type of structure and loading in which the principal stresses can be computed at the center of the opening.

Both methods are approximate and are limited to openings at least three times the radius of the opening from the nearest boundary of the structure.

Note.—The paper by I. K. Silverman, Jun. Am. Soc. C. E., was published in November, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1937, by Messrs. R. D. Mindlin, and Chesley J. Posey.

[16] Director, Photoelastic Laboratory, U. S. Bureau of Reclamation, Denver, Colo.

[16a] Received by the Secretary January 20, 1937.

[17] *Technical Memorandum No. 457*, U. S. Bureau of Reclamation.

In fact, the author's formulas may be restricted to even greater distanc
since he assumes that $\frac{r_i}{r_o} = 0$ in arriving at Equations (47), (48), and (49
It is evident that the circle, $\rho = r_o$ must lie within the structure, whic
implies that the opening must be many times r_i removed from the boundarie
The region of validity should be definitely stated by the author. An approx
mate theory is of little value in practice without a stipulation of the limi
of validity.

If the opening is dangerously close to the boundaries of the structur
or if its shape deviates too much from a circle, it is customary at th
Bureau of Reclamation to check the approximate analysis by photo-elasti
experimentation.[18]

It has been stated that the author's approach is somewhat more laborio
than the methods now in vogue, but even so, it has great merit if it i
used only as a check on these other methods. It would certainly be
great interest if Mr. Silverman would compare numerical results obtained b
each method under identical conditions. In doing this the tensile stresse
and the necessary steel reinforcement should be computed over the maximu
line, which, in the author's example, is seen by Fig. 4 to occur at abou
$\theta = +83°$ and $\theta = -103°$, and not at $\theta = \pm 90$ degrees.

A source of uncertainty in the results obtained by both methods
approach is the change in behavior of the structure caused by the rei
forcement steel (if such is needed). This question has been discussed by th
writer,[17] including the effect of temperature changes and a uniform pressu
acting in the circular opening.

In publishing solutions of elastic two-dimensional problems many write
consider it necessary to repeat the general theory of the Airy-Maxwell stre
function, including the equilibrium equations, stress definitions, and th
differential equation which expresses compatibility in accordance with Hooke
generalized stress-strain relations. This seems a needless waste of spac
inasmuch as many good references are available. On the other hand, importa
steps are often omitted which would clarify the situation for the inexperience
reader and facilitate checks on the mathematical procedure.

In the present case the process of transforming the stress functions t
the new origin $(x_0\, y_0)$ should be given in some detail, because it is of fund
mental importance and of general use, and no reference is readily available i
case body forces are involved. In manipulating stress functions which repre
sent body forces (weight, inertia, percolation), one soon discovers that it i
"tricky" and full of pitfalls.

Change of Co-Ordinates.—The author does not state the definitions
stresses in rectangular components, when body forces, c_1 and c_2, are presen
It is here to be noted that more than one stress definition is possible, but i
the stress function shall have the same form in rectangular and polar co-ord
nates, the writer has shown[19] that the stresses must be defined as follows i

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[18] *Technical Memorandum No. 451,* U. S. Bureau of Reclamation.

[19] *Bulletin,* Am. Math. Soc., June, 1934, p. 427.

rectangular components:

$$s_x = \frac{\partial^2 F}{\partial y^2} - c_1\, x - c_2\, y \qquad (62a)$$

$$s_y = \frac{\partial^2 F}{\partial x^2} - c_1\, x - c_2\, y \qquad (62b)$$

and,

$$s_{xy} = -\frac{\partial^2 F}{\partial x\ \partial y} \qquad (62c)$$

By comparing the author's function, Equation (22), with the function given by the writer[20] for the same case, it may be inferred that the author's stress definitions in rectangular components, are as in Equations (62).

If the origin is now chosen at a point $(x_0\, y_0)$ and if the new axes $(\xi\, \eta)$ are parallel with the original axes $(x\, y)$, then:

$$x = \xi + x_0 \qquad (63a)$$

and

$$y = \eta + y_0 \qquad (63b)$$

Substituting these values in Equations (62), the stress definitions in the new co-ordinate system become:

$$s_\xi = \frac{\partial^2 F'_g}{\partial \eta^2} - c_1\,(\xi + x_0) - c_2\,(\eta + y_0) \qquad (64a)$$

$$s_\eta = \frac{\partial^2 F'_g}{\partial \xi^2} - c_1\,(\xi + x_0) - c_2\,(\eta + y_0) \qquad (64b)$$

and,

$$s_{\xi\eta} = -\frac{\partial^2 F'_g}{\partial \xi\ \partial \eta} \qquad (64c)$$

in which F'_g is obtained simply by substituting Equations (63) into the expression for F_g. In the present case, F_g is given by Equation (22), so that:

$$F'_g = \frac{1}{6}\left[c_1\,(\xi + x_0)^3 + (c_1\, k + c_2 - 2\, c_2\, k^2_2\,(\eta + y_0)^3\right]$$
$$+ 3\, c_2\, k_2\,(\xi + x_0)\,(\eta + y_0)^2 \qquad (65)$$

The stresses are now obtained by Equations (64). If, in general, F' is defined as:

$$F' = F_g' - \frac{1}{2}\,(c_1\, x_0 + c_2\, y_0)\,(\xi^2 + \eta^2) \qquad (66)$$

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[20] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), Equation (21), p. 1248.

then the stress definitions in the new co-ordinate system may again be expressed in the same general form as Equations (62), namely:

$$s_\xi = \frac{\partial^2 F'}{\partial \eta^2} - c_1\, \xi - c_2\, \eta \ldots\ldots\ldots\ldots\ldots\ldots (67a)$$

$$s_\eta = \frac{\partial^2 F'}{\partial \xi^2} - c_1\, \xi - c_2\, \eta \ldots\ldots\ldots\ldots\ldots\ldots (67b)$$

and,

$$s_{\xi\eta} = - \frac{\partial^2 F'}{\partial \xi\ \partial \eta} \ldots\ldots\ldots\ldots\ldots\ldots (67c)$$

in which F'' is given by Equation (66). The polar stress components are given by Equations (20). It is important to note that in order to retain the general stress definitions, Equations (62) or Equations (67), it is necessary to subtract the term, 0.5 $(c_1\, x_0 + c_2\, y_0)$ $(\xi^2 + y^2)$, from the transformed function, F''_g. When no body forces are involved (that is when $c_1 = c_2 = 0$), the transformation is direct.

If the co-ordinate system is to be rotated through the angle, α, about the origin merely replace θ by $\theta + \alpha$ in Equations (20).

Compatibility.—Mr. Silverman makes use of St. Venant's principle when he states that the stresses in the original structure acting on the circle, $\rho = r_o$, are unaffected by the existence of the opening, if r_o is large compared with r_i. This principle is somewhat vague, but has been found to hold in many exact solutions and experiments, if applied properly. If r_o falls inside the structure, it is believed that the principle applies in the present case. This then enables the author to ignore compatibility between the regions inside and outside the circle, $\rho = r_o$, so that he only needs to equate the stresses coming from the inside and the outside regions. However, in setting up Stress System II (Equations (34)), in the annular ring, $\rho = r_o \rightarrow r_i$, due to body forces, he makes use of a stress function of the type, $\theta\ \rho \sin \theta$, which gives multi-valued displacements if the origin is entirely surrounded by elastic material. This is not permissible, of course, because it implies that the annular ring has a slit connecting the inside with the outside region. The edges of this slit will be dislocated and must be brought together again by another stress function, giving rise to what is generally known as "dislocation stresses." These are functions of the elastic properties of the material.[21] The complete function which must be used for the author's Stress System II has been presented by the writer.[20]

If no pressure acts on the inside of the hole, this function may be written, in the author's notation:

$$A \left\{ \frac{1}{\rho} + \frac{2\, \rho \log_e \rho}{r^2_i} \right\} \cos (\theta - \beta) + \frac{r^2_i}{2}\, c\, \rho\, \theta \sin (\theta - \beta) \ldots\ldots (68)$$

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[21] *Proceedings*, Math. Soc., London, Vol. 30, 1900, p. 103; and, also, "The Mathematical Theory of Elasticity", by A. E. H. Love, 1927, p. 221.

The stresses are defined by Equations (20). The constant, A, in Equation (68) must be determined so that displacements are single valued; that is, it must be unchanged if θ is replaced by $\theta + 2\,n\,\pi$, in which n is an arbitrary integer. It will be found that,

$$A = -\frac{1}{8}\,r^4_i\,c\,(1-\mu)\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(69)$$

will make the displacements single valued.[22] It is to be noticed that, in Equation (69) in which μ is Poisson's ratio, A is a function of this elastic property, μ.

The Stress System II is now found by applying Definitions (20) to Function (68):

$$s_\rho = -\frac{r_i}{4}\,c\,(1-\mu)\left(\frac{r_i}{\rho} - \frac{r^3_i}{\rho^3}\right)\cos(\theta-\beta) + \frac{r^2_i}{\rho}\,c\cos(\theta-\beta) - \rho\,c\cos(\theta-\beta)\ldots\ldots\ldots(70a)$$

$$s_\theta = -\frac{r_i}{4}\,c\,(1-\mu)\left(\frac{r_i}{\rho} + \frac{r^3_i}{\rho^3}\right)\cos(\theta-\beta) - \rho\,c\cos(\theta-\beta)\,..(70b)$$

and,

$$s_{\rho\theta} = -\frac{r_i}{4}\,c\,(1-\mu)\left(\frac{r_i}{\rho} - \frac{r^3_i}{\rho^3}\right)\sin(\theta-\beta)\;\ldots\ldots\ldots(70c)$$

It will be seen that s_ρ and $s_{\rho\theta}$ vanish over the circle, $\rho = r_i$; and, that s_ρ and s_θ become equal to $-\,r_o\,c\cos(\theta-\beta)$ and $s_{\rho\theta} = 0$ over the circle, $\rho = r_o$, if $\frac{r_i}{r_o}$ approaches zero, thus satisfying the author's conditions.

Attention should be called to the fact that in the writer's use of Function (68) for the determination of stresses in a fillet of a dam, the constant, A, is arbitrary because the origin is not completely surrounded by elastic material in the case of a fillet, that is, is always less than 2π. (In this connection compare the methods of finding displacements given elsewhere by the writer[21].)

The magnitude of Stress System II is usually negligible in the regions of validity of the author's formulas so that it could safely be ignored entirely. The reason for bringing to light the correction in System II is that an important principle is involved in the general application of stress functions giving multiple-valued displacements for the determination of stresses due to temperature variations and body forces in structures having doubly connected regions.[23]

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[22] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1246; also, "Photo-Elasticity", by Coker and Filon, 1931, p. 327.

[23] *Journal of Applied Mechanics*, Biot, A. S. M. E., June, 1935.

V. L. FEDOROV,[24] ESQ. (by letter).[24a]—The study of the stress distribution near the holes in dams and buttresses is interesting because it may help to discover the cause of cracks, that often originate in inspection galleries. The solution offered by Mr. Silverman is correct only in the case of water load; for the case of the action of mass load it should be modified as outlined in this discussion.[25]

When a circular ring is cut from a massif subjected to internal strain due to its own weight, the resultant stress on the outer face of the hole is not zero, but equal to the weight of the ring. In this case the stress distribution depends on the elastic constants of the material, which fact was discovered by Michell.

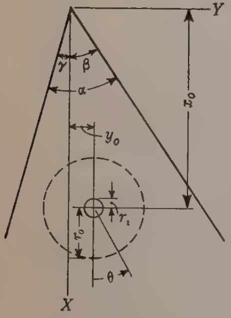

FIG. 7.

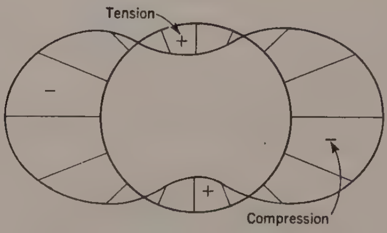

FIG. 8.

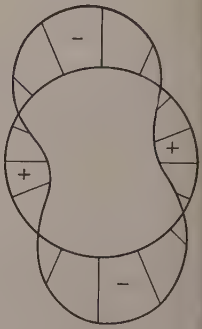

FIG. 9

Referring to Fig. 7, assume a ring in which r_o is so much greater than r_i that $\frac{r_i}{r_o}$ may be assumed equal to zero for all practical purposes. The stresses are then expressed by:

$$s_{\rho o} = c\Big[A_0 + (A_1 - 1)\, r_o \cos\theta + B_1\, r_o \sin\theta + A_2 \cos 2\,\theta + B_2 \sin 2\,\theta + A_3\, r_o \cos 3\,\theta + B_3\, r_o \sin 3\,\theta\Big] \quad \ldots\ldots\ldots\ldots\ldots (71a)$$

and,

$$s_{so} = c\Big[A_1\, r_o \sin\theta - B_1\, r_o \cos\theta - A_2 \sin 2\,\theta + B_2 \cos 2\,\theta - A_3\, r_o \sin 3\,\theta + B_3\, r_o \cos 3\,\theta\Big] \quad \ldots\ldots\ldots\ldots\ldots (71b)$$

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[24] Dozent, Industrial Inst.; Senior Scientific Worker, All-Union Scientific Research Inst. of Hydrotecknik, Leningrad, Union of Socialistic Soviet Republics.

[24a] Received by the Secretary January 21, 1937.

[25] Based upon a thesis (in Russian) entitled "The Influence of the Round Hole on the Distribution of Stresses in a Dam"; submitted by the writer in 1935 to the State Commission of the Industrial Inst., Leningrad, U. S. S. R., in partial fulfillment of the requirements for the degree of Candidate of Technical Sciences.

in which c = weight per unit volume,

$$A_o = \frac{1}{2}\left[\frac{\sin(\beta - \gamma)\cos\alpha}{\sin^2\alpha}\, y_o - \left(1 - \frac{2\sin\gamma\sin\beta\cos\alpha}{\sin^2\alpha}\right) x_o\right] \qquad (72a)$$

$$A_1 = \frac{1}{4}\left(1 + \frac{2\sin\gamma\sin\beta\cos\alpha}{\sin^2\alpha}\right) \qquad (72b)$$

$$B_1 = \frac{\sin(\beta - \gamma)\cos\alpha}{4\sin^2\alpha} \qquad (72c)$$

$$A_2 = \frac{1}{2}\left[\frac{\sin 2(\beta - \gamma)}{2\sin^2\alpha}\, y_o - \left(1 - \frac{2\sin\gamma\sin\beta\cos(\beta - \gamma)}{\sin^2\alpha}\right) x_o\right] \qquad (72d)$$

$$B_2 = \frac{\sin\gamma\sin\beta}{\sin^2\alpha}\left[x_o\sin(\beta - \gamma) - 2\, y'_o\cos\gamma\cos\beta\right] \qquad (72e)$$

$$A_3 = \frac{1}{4}\left[\frac{2\sin\gamma\sin\beta}{\sin^2\alpha}(\sin\gamma\sin\beta + 3\cos\gamma\cos\beta) - 1\right] \qquad (72f)$$

and,

$$B_3 = \frac{\sin(\beta - \gamma)}{4\sin^2\alpha}(\cos\gamma\cos\beta + 3\sin\gamma\sin\beta) \qquad (72g)$$

Thus, the stresses in the ring, subjected to load, as determined by Equations (71), are distributed along the outer face (r_o in Fig. 7); there is no load along the inner face, r_i. The stress distribution in this ring may be solved by means of the stress function:

$$\begin{aligned}\phi = c\Big[& a_o \log\rho + l_o\rho^2 \\ & + \frac{a_1}{2}\rho\,\theta\sin\theta + (l_1\rho^3 + a'_1\rho^{-1} + l'_1\rho\log\rho)\cos\theta \\ & - \frac{c_1}{2}\rho\,\theta\cos\theta + (d_1\rho^3 + c'_1\rho^{-1} + d'_1\rho\log\rho)\sin\theta \\ & + (a_2\rho^2 + l_2\rho^4 + a'_2\rho^{-2} + l'_2)\cos 2\theta \\ & + (c_2\rho^2 + d_2\rho^4 + c'_2\rho^{-2} + d'_2)\sin 2\theta \\ & + (a_3\rho^3 + l_3\rho^5 + a'_3\rho^{-3} + l'_3\rho^{-1})\cos 3\theta \\ & + (c_3\rho^3 + d_3\rho^5 + c'_3\rho^{-3} + d'_3\rho^{-1})\sin 3\theta\Big] \qquad (73)\end{aligned}$$

The stresses are:

$$s_\rho = \frac{1}{\rho}\frac{\partial \phi}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \phi}{\partial \theta^2} - c\,\rho \cos\theta = c\left[\frac{a_0}{\rho^2} + 2\,l_0\right.$$
$$+ \left(\frac{a_1 + l'_1}{\rho} + 2\,l_1\,\rho - \frac{2\,a'_1}{\rho^3} - \rho\right)\cos\theta + \left(\frac{c_1 + d'_1}{\rho} + 2\,d_1\,\rho - \frac{2\,c'_1}{\rho^3}\right)\sin\theta$$
$$+ \left(2\,a_2 + \frac{6\,a'_2}{\rho^4} + \frac{4\,l'_2}{\rho^2}\right)\cos 2\,\theta - \left(2\,c_2 + \frac{6\,c'_2}{\rho^4} + \frac{4\,d'_2}{\rho^2}\right)\sin 2\,\theta$$
$$- \left(6\,a_3\,\rho + 4\,l_3\,\rho^3 + \frac{12\,a'_3}{\rho^5} + \frac{10\,l'_3}{\rho^3}\right)\cos 3\,\theta$$
$$\left. - \left(6\,c_3\,\rho + 4\,d_3\,\rho^3 + \frac{12\,c'_3}{\rho^5} + \frac{10\,d'_3}{\rho^3}\right)\sin 3\,\theta\right] \ldots\ldots\ldots (74a)$$

$$s_s = -\frac{\partial}{\partial \rho}\left(\frac{1}{\rho}\frac{\partial \phi}{\partial \theta}\right) = c\left[\left(\frac{l'_1}{\rho} + 2\,l_1\,\rho - \frac{2\,a'_1}{\rho^3}\right)\sin\theta - \left(2\,d_1\,\rho - \frac{2\,c'_1}{\rho^3} + \frac{d'_1}{\rho}\right)\cos\theta\right.$$
$$+ \left(2\,a_2 + 6\,l_2\,\rho^2 - \frac{6\,a'_2}{\rho^4} - \frac{2\,l'_2}{\rho^2}\right)\sin 2\,\theta - \left(2\,c_2 + 6\,d_2\,\rho^2 - \frac{6\,c'_2}{\rho^4} - \frac{2\,d'_2}{\rho^2}\right)\cos 2\,\theta$$
$$+ \left(6\,a_3\,\rho + 12\,l_3\,\rho^3 - \frac{12\,a'_3}{\rho^5} - \frac{6\,l'_3}{\rho^3}\right)\sin 3\,\theta$$
$$\left. - \left(6\,c_3\,\rho + 12\,d_3\,\rho^3 - \frac{12\,c'_3}{\rho^5} - \frac{6\,d'_3}{\rho^3}\right)\cos 3\,\theta\right] \ldots\ldots\ldots (74b)$$

and,

$$s_\theta = \frac{\partial^2 \phi}{\partial \rho^2} - c\,\rho\cos\theta = c\left[-\frac{a_0}{\rho^2} + 2\,l_0\right.$$
$$+ \left(6\,l_1\,\rho + \frac{2\,a'_1}{\rho^3} + \frac{l_1}{\rho} - \rho\right)\cos\theta + \left(6\,d_1\,\rho + \frac{2\,c'_1}{\rho^3} + \frac{d'_1}{\rho}\right)\sin\theta$$
$$+ \left(2\,a_2 + 12\,l_2\,\rho^2 + \frac{6\,a'_2}{\rho^4}\right)\cos 2\,\theta + \left(2\,c_2 + 12\,d_2\,\rho^2 + \frac{6\,c'_2}{\rho^4}\right)\sin 2\,\theta$$
$$+ \left(6\,a_2\,\rho + 20\,l_3\,\rho^3 + \frac{12\,a'_3}{\rho^5} + \frac{2\,l'_3}{\rho^3}\right)\cos 3\,\theta + \left(6\,c_2\,\rho + 20\,d_3\,\rho^3\right.$$
$$\left.\left. + \frac{12\,c'_3}{\rho^5} + \frac{2\,d'_3}{\rho^3}\right)\sin 3\,\theta\right] \ldots\ldots\ldots\ldots\ldots\ldots (74c)$$

The constant coefficients must be found from the boundary conditions: $\rho = r_i$; $s_\rho = 0$; and, $s_s = 0$. For $r = r_o$, when $\frac{r_i}{r_o} = 0$, Equations (74) become the same as Equations (71).

For terms containing sin 2 θ, cos 2 θ, sin 3 θ, and cos 3 θ, and for terms independent of θ, all coefficients may be found from the foregoing conditions, as follows: $a_0 = -A_0 \, r^2_i$; $l_0 = \frac{A_0}{2}$; $a_2 = -\frac{A_2}{2}$; $l_2 = 0$; $a'_2 = -\frac{A_2 \, r^4_i}{2}$; $l'_2 = A_2 \, r^2_i$; $c_2 = -\frac{B_2}{2}$; $d_2 = 0$; $c'_2 = -\frac{B_2}{2} r^4_i$; $d'_2 = B_2 \, r^2_i$; $a_3 = -\frac{A_3}{6}$; $l_3 = 0$; $a'_3 = -\frac{A_3 \, r^6_i}{3}$; $l'_3 = \frac{A_3 \, r^4_i}{2}$; $c_3 = -\frac{B_3}{6}$; $d_3 = 0$; $c'_3 = -\frac{B_3}{3} r^6_i$; and, $d'_3 = \frac{B_3 \, r^4_i}{2}$.

For terms containing cos θ and sin θ:

$$\frac{a_1 + l'_1}{r_i} + 2 \, l_1 \, r_i - \frac{2 \, a'_1}{r^3_i} - r_i = 0 \quad \text{.............. (75a)}$$

$$\frac{l'_1}{r_i} + 2 \, l_1 \, r_i - \frac{2 \, a'_1}{r^3_i} = 0 \quad \text{.................. (75b)}$$

$$\frac{a_1 + l'_1}{r_o} + 2 \, l_1 \, r_o - \frac{2 \, a'_1}{r^3_o} - r_o = (A_1 - 1) \, r_o \quad \text{......... (75c)}$$

$$\frac{l'_1}{r_o} + 2 \, l_1 \, r_o - \frac{2 \, a'_1}{r^3_o} = A_1 \, r_o \quad \text{................ (75d)}$$

$$\frac{c_1 + d'_1}{r_i} + 2 \, d_1 \, r_i - \frac{2 \, c'_1}{r^3_i} = 0 \quad \text{................ (75e)}$$

$$\frac{d'_1}{r_i} + 2 \, d_1 \, r_i - \frac{2 \, c'_1}{r^3_1} = 0 \quad \text{.................. (75f)}$$

$$\frac{c_1 + d'_1}{r_o} + 2 \, d_1 \, r_o - \frac{2 \, c_1'}{r^3_o} = B_1 \, r_o \quad \text{.............. (75g)}$$

and,

$$\frac{d'_1}{r_o} + 2 \, d_1 \, r_o - \frac{2 \, c'_1}{r^3_o} = B_1 \, r_o \quad \text{................ (75h)}$$

From Equations (75), it is possible to solve for $a_1 = r^2_i$ and $c_1 = 0$, leaving four equations (Equations (75*b*), (75*d*), (75*f*), and 75*h*)), with six unknowns for the solution of the remaining factors. In order that the displacements in the ring shall be single-valued in the case of plane strain, it is necessary to assume[26] that,

$$l'_1 = -\frac{1 - 2 \, \mu}{4 \, (1 - \mu)} \, a_1 \quad \text{.................... (76a)}$$

and,

$$d'_1 = -\frac{1 - 2 \, \mu}{4 \, (1 - \mu)} \, c_1 \quad \text{.................... (76b)}$$

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[26] "Probleme Spannungsferteilung in ebenen Systemen", von Timpe, *Zeitschrift für Math. und Physik*, Bd. 52, 1905, p. 374.

in which μ is Poisson's ratio for the material. Then: $l'_1 = -\dfrac{1-2\,\mu}{4\,(1-\mu)}\, r^2_i$

$a'_1 = \dfrac{1}{2}\left[A_1 - \dfrac{1-2\,\mu}{4\,(1-\mu)}\right] r^4_i$; $l_1 = \dfrac{A_2}{2}$; $d'_1 = 0$; $c'_1 = \dfrac{B_1}{2}\, r^4_i$; and $d_1 = \dfrac{B_1}{2}$. .

Finally, the stresses are:

$$s_\rho = c\left\{A_0\left(1 - \frac{r^2_i}{\rho^2}\right) + (A_1 \cos\theta + B_1 \sin\theta)\left(\rho - \frac{r^4_i}{\rho^3}\right)\right.$$
$$+ \left[\frac{1-2\,\mu}{4\,(1-\mu)}\left(\frac{r^4_i}{\rho^3} - \frac{r^2_i}{\rho}\right) + \frac{r^2_i}{\rho} - \rho\right]\cos\theta$$
$$+ (A_2 \cos 2\,\theta + B_2 \sin 2\,\theta)\left(1 + \frac{3\,r^4_i}{\rho^4} - \frac{4\,r^2_i}{\rho^2}\right)$$
$$\left. + (A_3 \cos 3\,\theta + B_3 \sin 3\,\theta)\left(\rho + \frac{4\,r^6_i}{\rho^5} - \frac{5\,r^4_i}{\rho^3}\right)\right\} \ldots\ldots\ldots (77a)$$

$$s_s = c\left\{(A_1 \sin\theta - B_1 \cos\theta)\left(\rho - \frac{r^4_i}{\rho^3}\right) + \frac{1-2\,\mu}{4\,(1-\mu)}\left(\frac{r^4_i}{\rho^3} - \frac{r^2_i}{\rho}\right)\sin\theta\right.$$
$$- (A_2 \sin 2\,\theta - B_2 \cos 2\,\theta)\left(1 - \frac{3\,r^4_i}{\rho^4} + \frac{2\,r^2_i}{\rho^2}\right)$$
$$\left. - (A_3 \sin 3\,\theta - B_3 \cos 3\,\theta)\left(\rho - \frac{4\,r^6_i}{\rho^5} + \frac{3\,r^4_i}{\rho^3}\right)\right\} \ldots\ldots (77b)$$

and,

$$s_\theta = c\left\{A_0\left(1 + \frac{r^2_i}{\rho^2}\right) + (A_1 \cos\theta + B_1 \sin\theta)\left(3\,\rho + \frac{r^4_i}{\rho^3}\right)\right.$$
$$- \left[\frac{1-2\,\mu}{4\,(1-\mu)}\left(\frac{r^2_i}{\rho} + \frac{r^4_i}{\rho^3}\right) + \rho\right]\cos\theta$$
$$- (A_2 \cos 2\,\theta + B_2 \sin 2\,\theta)\left(1 + \frac{3\,r^4_i}{\rho^4}\right)$$
$$\left. - (A_3 \cos 3\,\theta + B_3 \sin 3\,\theta)\left(\rho + \frac{4\,r^6_i}{\rho^5} - \frac{r^4_i}{\rho^3}\right)\right\} \ldots\ldots\ldots (77c)$$

The foregoing solution may be applied to gravity dams. In the case of hollow dams, the buttresses are considered as thin plates, instead of $\dfrac{1-2\,\mu}{4\,(1-\mu)}$ (see Equation (76a), one must substitute $\dfrac{1-\mu}{4}$.

To demonstrate the characteristics of the results, the hoop stresses around a hole due to mass load in a gravity dam are represented in Fig. 8. There is tension along the upper and lower regions, and compression along the sides. The water load for this dam yields the hoop stress distribution shown in Fig. 9.

F. W. HANNA,[27] M. AM. SOC. C. E. (by letter).[27a]—A subject that is very important in the design of dams, is described in this paper. That both longitudinal and transverse cracks appear in the inspection and drainage galleries of most dams is to be expected; and, owing to paucity of data on the subject, the designers of dams are in need of technical information on their cause and prevention. Such technical data are provided, in a large measure, by the author. He is to be congratulated therefore, not only on having introduced the subject, but on having presented a careful and laborious analysis of the particular phase of it involved in buttress dams.

The solutions presented are long and the resulting formulas involved and cumbersome. Some of these troubles could have been avoided by taking the origin of co-ordinates at the center of the circular opening initially and using the computed pressures at this point as constant for limited areas around it, the computed pressures being predicated on the absence of the hole in the buttress. This would have involved some

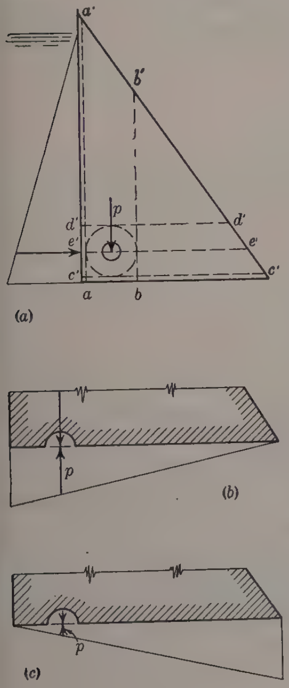

FIG. 10.

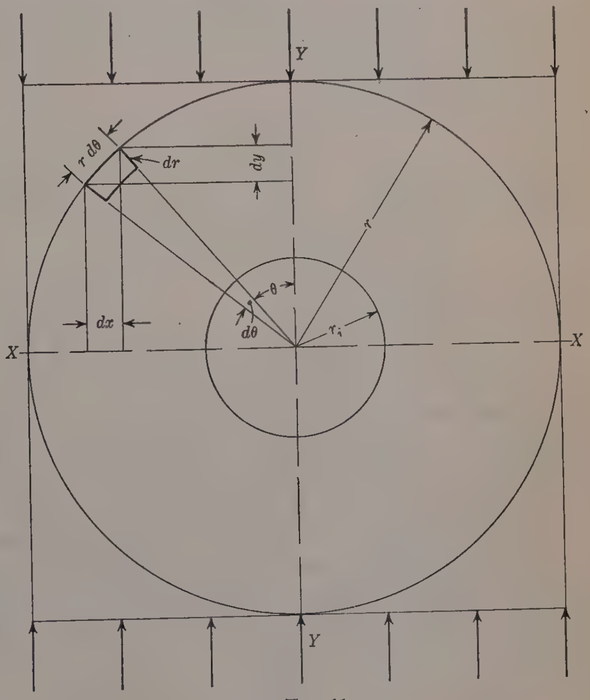

FIG. 11.

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[27] Cons. Engr., Ankeny, Iowa.

[27a] Received by the Secretary February 15, 1937.

approximation, but not sufficient, in a high dam with the opening near the base, to vitiate the results.

Approximate Formulas for Circular Holes.—The writer suggests the use of an approximate solution of the problem of stresses caused by holes in a dam. Let Fig 10(*a*) represent a section of the theoretical profile of a gravity dam with a circular hole through it near the base to serve as a drainage and inspection gallery. Let p be the computed vertical mass pressure per square foot at the center of the hole calculated on the basis of a solid dam. Consider the section, $aa'\ b'b$, as a plate, 1 ft thick, perpendicular to the paper, acted on by a uniform unit vertical pressure of p in the vicinity of the hole as indicated in Fig. 10(*b*). The vertical pressure, p, will be a maximum for reservoir empty, as shown by a comparison of Fig. 10(*b*), with Fig. 10(*c*) showing the vertical pressure, p, for reservoir full.

Referring to Fig. 11, let it be asumed that the disturbed stresses lie mainly within the larger circle shown thereon. The stresses at the boundary of the outer circle are essentially the same as in the dam without the hole. The radial stress, s_r, perpendicular to the elementary volume will be $s_r\,r\,d\theta$ and will be balanced by the vertical force, $p\,dx$, resolved to the opposite side of this volume; therefore, $s_r\,r\,d\theta = p\,dx\cos\theta$, whence $s_r = p\cos\theta\,dx \div r\,d\theta$; therefore,

$$s_r = p\cos^2\theta = \frac{p}{2} + \frac{p}{2}\cos 2\,\theta \quad\ldots\ldots\ldots\ldots(78)$$

In like manner the shearing stress, s_s, would be opposed by the force,

$$s_s = -\,0.5\,p\sin 2\,\theta \quad\ldots\ldots\ldots\ldots(79)$$

and the tangential stress is,

$$s_\theta = p\sin^2\theta = \frac{p}{2} - \frac{p}{2}\cos 2\,\theta \quad\ldots\ldots\ldots\ldots(80)$$

The stress distribution may now be considered as consisting of two parts, one the constant components, $0.5\,p$, and the other the components, $0.5\,p\cos 2\,\theta$, and the shearing force, $0.5\,p\sin 2\,\theta$, both containing functions of θ. The stresses due to these two groups of forces can best be determined separately and their sum taken for their total effect.

The first group of forces act symmetrically around the center of the hole in the dam and is represented by the stress distribution in a thick hollow cylinder subjected to external normal pressure. By adding Equation (78) and Equation (80),

$$s'_r - s'_\theta = p\ (\text{a constant}) \quad\ldots\ldots\ldots\ldots(81)$$

Now, from Fig. 11, $(s'_r + ds'_r)\,(r + dr) - s'_r = s'_\theta\,dr$, by equilibrating the acting pressure and the resisting stress. Dropping differentials of higher orders,

$$r\,ds'_r + s'_r\,dr = s'_\theta\,dr \quad\ldots\ldots\ldots\ldots(82)$$

Solving for s_θ in Equation (81), and substituting the resulting value in Equation (82),

$$r\,ds'_r + s'_r\,dr = (p - s'_r)\,dr \quad \text{.....................(83)}$$

Re-arranging,

$$\frac{r\,ds'_r}{s'_r - \frac{p}{2}} = -\frac{2\,dr}{r} \quad \text{..................... (84)}$$

Integrating,

$$\log\left(s'_r - \frac{p}{2}\right) = \log\frac{C}{r^2} \quad \text{...................(85)}$$

Finding the anti-logs of both sides of Equation (85),

$$s'_r - \frac{p}{2} = \frac{C}{r^2} \quad \text{.........................(86)}$$

When $r = r_1$ in Equation (86), $s'_r = 0$ and, therefore,

$$-\frac{p}{2} = \frac{C}{r^2} \quad \text{......................... (87)}$$

Solving for C in Equation (87),

$$C = -\frac{p\,r^2_1}{2} \quad \text{......................... (88)}$$

Substituting the value of C just found,

$$s'_r - \frac{p}{2} = -\frac{p\,r^2_1}{2\,r^2} \quad \text{..................... (89)}$$

Solving for s'_r in Equation (89), the following important equation is derived,

$$s'_r = \frac{p}{2}\left(1 - \frac{r^2_1}{r^2}\right) \quad \text{.....................(90)}$$

Eliminating s'_r between Equation (89) and Equation (81) and solving for s'_θ, the companion important equation is found,

$$s'_\theta = \frac{p}{2}\left(1 + \frac{r^2_1}{r^2}\right) \quad \text{.....................(91)}$$

Since functions of θ are not involved in this group of forces, a third important result is obtained,

$$s'_s = 0 \quad \text{..............................(92)}$$

Addressing the computations to the solution for the stresses produced by the second group of forces, involving functions of θ, let the Airy stress function, F, be taken as,

$$F = f\,(r) \cos 2\,\theta \quad \text{.......................} (93)$$

since F is a function of r and θ. Differentiating Equation (93) twice with respect to θ and noting that the result must hold for $\theta = 0$ as well as for all other values around the center of the hole, there results the equation,

$$\frac{d^2 F}{r^2\, d\,\theta^2} = -\frac{4\,f\,(r)}{r^2} \quad \text{.......................} (94)$$

Substituting this value in Equation (21), of the paper, the ordinary differential equation becomes,

$$\left(\frac{d^2}{dr^2} + \frac{d}{r\,dr} - \frac{4}{r^2}\right)\left(\frac{d^2 f}{dr^2} + \frac{df}{r\ dr} - \frac{4\,f}{r^2}\right) = 0 \quad \text{........} (95)$$

The general solution of this equation, as can be determined by checking back on Equation (94), is,

$$f\,(r) = C_1\,r^2 + C_2\,r^4 + \frac{C_3}{r^2} + C_4 \quad \text{.................} (96)$$

Substituting the value of $f\ (r)$ in Equation (93) there results the following stress function equation,

$$F = \left(C_1\,r^2 + C_2\,r^4 + \frac{C_3}{r^2} + C_4\right) \cos 2\,\theta \quad \text{.............} (97)$$

The last term in each of the author's Equations (20) may be dropped due to assumed constancy of weight in the vicinity of the hole. Then these equations become, respectively,

$$s''_r = \frac{d^2 F}{r^2\ d\,\theta^2} + \frac{dF}{r\,dr} \quad \text{.......................} (98a)$$

$$s''_\theta = \frac{d^2\ F}{dr^2} \quad \text{............................} (98b)$$

and,

$$s''_s = \frac{d}{dr}\left(\frac{dF}{r\ d\,\theta}\right) \quad \text{......................} (98c)$$

These are the stress components to be found from the use of Equation (97) and the boundary conditions of the problem. Performing the differentiations on Equation (97) as required by Equations (98) and substituting the results in these equations:

$$s''_r = -\left(2\,C_1 + \frac{6\,C_3}{r^4} + \frac{4\,C_4}{r^2}\right)\cos 2\,\theta \ldots\ldots\ldots\ldots (99a)$$

$$s''_\theta = \left(2\,C_1 + 12\,C_2\,r^2 + \frac{6\,C_3}{r^4}\right)\cos 2\,\theta \ldots\ldots\ldots\ldots (99b)$$

and,

$$s''_s = \left(2\,C_1 + 6\,C_2\,r^2 - \frac{6\,C_3}{r^4} - \frac{2\,C_4}{r^2}\right)\sin 2\,\theta \ldots\ldots\ldots (99c)$$

The constants are to be determined from the external boundary conditions at a distance as required by Equations (78) (79) and (80), from the fact that the stresses are zero at the edge of the hole, and from the fact that $\frac{r_1}{r} = 0$, approximately, for distant points. Applying these principles and solving the constants it will be found that: $C_1 = -\frac{p}{4}$; $C_2 = 0$; $C_3 = -\frac{p\,r_1^4}{4}$ and $C_4 = \frac{p\,r_1^2}{2}$. Substituting these values in Equations (90):

$$s''_r = \frac{p}{2}\left(1 + \frac{3\,r_1^4}{r^4} - \frac{4\,r_1^2}{r^2}\right)\cos 2\,\theta \ldots\ldots\ldots\ldots (100a)$$

$$s''_\theta = -\frac{p}{2}\left(1 + \frac{3\,r_1^4}{r^4}\right)\cos 2\,\theta \ldots\ldots\ldots\ldots\ldots (100b)$$

and,

$$s''_s = -\frac{p}{2}\left(1 - \frac{3\,r_1^4}{r^4} + \frac{2\,r_1^2}{r^2}\right)\sin 2\,\theta \ldots\ldots\ldots\ldots (100c)$$

Now, adding Equations (100) to Equations (90), (91), and (92), respectively, and noting that $s'_r + s''_r = s_r$, $s'_\theta + s''_\theta = s_\theta$, and, $s'_s + s''_s = s_s$, the following final important equations are obtained,

$$s_r = \frac{p}{2}\left[1 - \frac{r_1^2}{r^2} + \left(1 - \frac{4\,r_1^2}{r^2} + \frac{3\,r_1^4}{r^4}\right)\right]\cos 2\,\theta \ldots\ldots (101a)$$

$$s_\theta = \frac{p}{2}\left[1 + \frac{r_1^2}{r^2} - \left(1 + \frac{3\,r_1^4}{r^4}\right)\cos 2\,\theta\right] \ldots\ldots\ldots\ldots (101b)$$

and,

$$s_s = -\frac{p}{2}\left(1 + \frac{2\,r_1^2}{r^2} - \frac{3\,r_1^4}{r^4}\right)\sin 2\,\theta \ldots\ldots\ldots\ldots (101c)$$

Equations (101) give, respectively, the radial compression, the tangential tension, and the radial shear in the vicinity of the hole. If the value of r is very large, s_r, s_s, and s_θ approach the values given in Equations (78), (79), and (80). If $r = r_i$, $s_r = s_s = 0$, and $s_\theta = p - 2\,p \cos 2\,\theta$. For $\theta = 90°$ and $270°$, $s_\theta = 3\,p$, the maximum compression at the ends of the horizontal diameter. For $\theta = 0°$ and $180°$, $s_\theta = -\,p$, the maximum tension at the end of the vertical diameter. In general, the compressive stresses are not serious in a dam although they may be quite large, but the tensional stresses at the top and bottom of the hole should be provided for with appropriate steel reinforcement.

Formulas similar to Equations (102) have been developed by several investigators[28] for tension in thin plates. In this discussion they have been developed for compression in thin plates in order to make them apply to dams. Such an application has been made previously.[29]

Approximate Formulas for Elliptical Holes.—Drainage and inspection galleries are not often circular in shape, but generally may be made to approach ellipses. By making the same assumptions as to pressures and as to the use of vertical and horizontal strips of the dam as plates, stress formulas may be developed for elliptical holes.

The development of formulas for the determination of stresses caused by elliptical holes in dams involves the use of curvilinear co-ordinates and hyperbolic functions. Formulas have been developed by Inglis for elliptical plates[30] and they may be applied to holes in dams by making the same assumptions as previously made herein for circular holes. The resultant Inglis formulas applicable to the case are not included herein as the reader can find their development in available engineering literature.

Square Holes.—In dams square holes should be avoided, especially with sharp corners. Rounding the corners with liberal radia helps the stress conditions. The injunction against the use of square holes applies to the customary flat bottoms in otherwise well designed drainage and inspection galleries.

Limitation of Approximate Formulas.—It is appreciated that the foregoing approximate formulas are not applicable where there is too much variation in the pressure in the vicinity of the hole in the dam, which is the case in a low dam. However, inspection and drainage galleries are not often needed in such structures. In high dams where such openings are needed, the pressures are fairly uniform and only slight error is involved in the assumption that uniformity of pressure exists in the vicinity of the hole both for mass and water pressure. It should be observed that mass pressure is predominant and is at a maximum with the absence of water pressure. When the reservoir is full, nearly all the mass pressure disappears and the horizontal water pressure predominates.

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[28] *Zeitschrift* des Vereines Deutscher Ingenieure, Vol. 42, 1898: "Stresses in Plates Having Discontinuities and Some Problems Connected with Them", by K. Suyehiro, *Engineering*, September 1, 1911; "Drang and Zwang", Vol. 1, pp. 314–319, and "Vorlesungen Uber Technische Mechaniks", by A. Föppel, Vol. V, p. 352; and by "Theory of Elasticity", by S. Timoshenko, pp. 75–78.

[29] "The Design of Dams", by Hanna and Kennedy, pp. 160–162.

[30] "Stresses in a Plate Due to the Presence of Cracks and Sharp Corners", by C. E. Inglis, *Transactions*, Inst. of Naval Archts., 1913, Pt. I, pp. 219–230.

Little valid objection can be made to the assumption that a narrow vertical slice of a gravity dam can be considered as a plate. Gravity dam sections are designed on the basis of the existence of such solitary plates, and this in spite of the effects of Poisson's ratio on their sides. Corrections can be made for the effects of this ratio on the sides of the slice by making a separate computation for a stress equal to 25% of the major force. The effect of Poisson's ratio is to lessen that of the major force.

Cracks in Holes in Dams.—As far as the writer is aware, the Pardee Dam was the first in which reinforcement steel was placed around the drainage and inspection gallery to provide against tension. This gallery is 5 ft wide and 10 ft high, circular at the top and flat at the bottom. The floor of the main gallery is 300 ft below the top of the dam, the average mass pressure, p, being equal to 43 200 lb per sq ft at the center of the gallery.

The equation for the total horizontal tensile force at the top of the gallery, on the assumption that it is circular, is given by[31]:

$$F = -\frac{p\,r_t}{3\sqrt{3}} \quad \text{......................} \quad (102)$$

From Equation (102), based on a radius of $2\frac{1}{2}$ ft, the total horizontal tensile stress at the top of gallery of the Pardee Dam is 20 800 lb. The reinforcement consists of 1-in. square bars placed 1 ft apart, extending entirely around the opening. Nevertheless, there is a hair-crack along the top of the gallery for the entire length. No visible cracks exist at the corners of the flat floor, where they might be expected to be found.

The vertical compressive stress at the ends of the horizontal diameter, as given by Equation (101*b*), is $3\,p = 3 \times 43\,200 = 129\,600$ lb per sq ft, or about 900 lb per sq in., which is not excessive.

If the gallery in the Pardee Dam is assumed to be elliptical with the major axis vertical, and equal to twice the minor axis, the opening in the dam will be more closely approximated. The unit tangential compressive stress at the ends of the horizontal diameter due to mass load is given by the formula,

$$s_\theta = p\left(1 + \frac{2\,b}{a}\right) \quad \text{......................} \quad (103a)$$

and the unit tensile stress at the ends of the vertical diameter is given by the formula.

$$s_\theta = -\,p \quad \text{........................} \quad (103b)$$

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[31] "The Design of Dams", by Hanna and Kennedy, p. 163.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

RECLAMATION AS AN AID TO INDUSTRIAL AND AGRICULTURAL BALANCE

Discussion

By Messrs. John P. Ferris, and L. C. Gray

John P. Ferris,[18] Esq. (by letter).[18a]—This paper states the case for an attempt to create a new industrial region to accompany a new agricultural region, which latter would normally be expected to come into existence as a result of a major reclamation development.

The National Industrial Conference Board reports[19] a startling displacement of employment in agriculture relative to other types of employment. In 1930, about 88 persons per thousand of the population of the United States were employed in agriculture, whereas two generations earlier (1870) there were about 180 persons per thousand thus employed. A variety of causes accounts for this astonishing shift, some of which the writer has discussed elsewhere.[20] As this displacement has occurred in existing agricultural regions it is natural that it has produced a large surplus rural population. In many counties in the rural Southeast this surplus appears to be from 400 to 1 800 workers.

The nation, therefore, faces a parallel problem to that discussed by the authors—the problem of the region that is being subjected to excessive drainage of both human and material resources. In such areas the outgo exceeds income, and the "books" are balanced by a continual depletion of basic resources and by increasing debt.

Migration is beset by great practical difficulties. A logical starting point for the diffusion of industrial employment into rural areas seems to be in some of the regions of increasing surplus rural population, where, at the same

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Note.—The paper by Ernest P. Goodrich and Calvin V. Davis, Members, Am. Soc. C. E., was published in November, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1937, by Joseph Jacobs, M. E. McIver, and C. S. Jarvis.

[18] Norris, Tenn.

[18a] Received by the Secretary February 10, 1937.

[19] "Gainful Workers per 1000 Population in the United States, 1870–1930", *Bulletin*, National Industrial Conference Board, Inc., August, 1932, Chart No. 283, Road Maps of Industry Series.

[20] "Industry, Agriculture, and Population Readjustment", *Mechanical Engineering*, 1936.

time, the new "framework" of roads and power can be made available. Of course, a number of the new water-shed projects are situated in such areas.

Surplus rural population results both from the large technological displacement in agriculture generally, and from the high birth rate in many rural areas; for instance, the farm population of the South, in the opinion of O. E. Baker,[21] Senior Agricultural Economist, Bureau of Agricultural Economics, U. S. Department of Agriculture, is multiplying at a rate which may result in its contributing approximately ten times as many people to the national total at the end of a century as an equal number of adults in the large cities.

In connection with this matter of population, the data of the California Water Project Authority seem to be open to question. Messrs. Warren S. Thompson and P. K. Whelpton, of the Scripps Foundation for Research in Population Problems,[22] estimate that in 1960 the population of California will be about 7 940 000, instead of 11 600 000 estimated by the Water Project Authority.

The writer is in hearty agreement with the authors concerning the desirability of diversification of employment. Many of the benefits, however, are achieved provided there is diversification within a small area; it is not necessarily most efficient for an individual to work both on the farm and in industry. Much can be said for an arrangement under which some member of a farm family spends his full time in farming, with industry and related rural community services absorbing the surplus workers from farms. This preserves the family farm system, which has many advantages, both in the matter of incentives and in the matter of security. Although many of the part-time farming or gardening projects to which Messrs. Goodrich and Davis refer, have been measurably successful, the fact that the land is worked by non-owners with no permanent security has caused many of these enterprises to be regarded by the participants as emergency relief, rather than as a way of living.

A third form of decentralization which has occurred in many parts of the country, results from highly developed transportation facilities and widespread availability of electric power in hundreds of the smaller towns. An increasing number of city workers with cash incomes are actually living on farms near industrial centers and "commuting" back and forth, the cost being divided among the several workers who ride in each car. Such families are supplementing their income from industry with some "live-at-home" income derived from small tracts of their own land.

Such programs as the foregoing, as well as the type of development proposed by Messrs. Goodrich and Davis, assume a new relation of industrial location to the national welfare. In the first stages of America's economic development, the basic soil economies shaped manufacturing developments. McCormick manufactured his reaper at Chicago, Ill., because this city was a commercial outpost in communication with the great wheat area, and because coal and steel could be shipped cheaply from Pennsylvania on the Great Lakes waterway, and, later, on the east-west railroads. Settlement of the industrially skilled immigrant populations from Europe naturally followed these same transportation routes.

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[21] Opinion expressed in 1936 in a letter to the writer.

[22] "Estimates of Future Population by States, 1935 to 1960", Oxford, Ohio.

Now, however, the country seems to be entering a new stage, one in which industry can exert a powerful guiding force in shaping the basic economies under which the people live. To-day, there are "network" systems of roads and electric power; many industries no longer find it necessary, as they did in the past, to huddle in large cities along the few old transportation "arteries." Industry can, to a much greater extent than ever before, now establish the location of its demand for the materials which it buys and thus influence the location of their production.

L. C. Gray,[23] Esq. (by letter).[23a]—According to the authors, advantages would accrue if industry were decentralized and small plants co-ordinated with agriculture in irrigated areas. Under this plan industrial workers in small factories would cultivate small tracts of land, either co-operatively or individually, and would produce at least part of their vegetable food requirements. Undoubtedly, certain advantages can be gained by proper co-ordination of agriculture and industry. However, the number and extent of benefits to be derived appear to be somewhat over-emphasized in this paper.

Both social and economic factors should receive consideration in judging the desirability of such a program. Social aspects are less tangible and less subject to measurement than economic factors, and the relative weight that should be given to them cannot be determined with precision. The value placed on certain social aspects by different individuals varies considerably. What one would consider highly desirable, another might consider undesirable. Because of this fact, the social advantages and disadvantages of decentralizing industry and integrating it with agriculture, have been omitted from this discussion.

The degree of economic success that can be attained by such integration will depend largely upon certain factors influencing the industrial enterprises and agricultural production. Sufficient quantities of raw products should be available under conditions which would allow successful competition with other manufacturers of similar products. In some of the irrigation projects of the West, a large amount of hydro-electric power is available at low rates. Such areas often offer an advantage to manufacturing industries which require large quantities of power, and in which the cost of power represents a large part of the total production cost. Under such conditions the manufacturing of non-agricultural products may assume considerable proportions. A local product can seldom, however, be expected to displace all similar products in near-by markets. If plants have a capacity greater than can be disposed of satisfactorily near-by, competition with more distant plants in other areas at higher transportation costs should be expected.

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[23] Asst. Administrator, Resettlement Administration, U. S. Dept. of Agriculture, Washington, D. C.

[23a] Received by the Secretary March 17, 1937.

Conditions should also be favorable to agriculture in the area, both for the production of any raw materials for manufacture and for garden crops. The cost of water in relation to crop values should be moderate. Soil should be fertile and of suitable texture for irrigation. Topography should be fairly level, and other agricultural and water factors should be favorable.

If agriculture and industry are to be co-ordinated according to some such plan on newly constructed projects, the development of the irrigation projects and industrial plants should take place simultaneously. Workers will depend upon industry for the major part of their income from the time they arrive on the project, and farmers will depend on it for marketing their crops. This situation may be difficult to bring about. Irrigation projects usually require heavy expenditures and large units become available for use at one time. If the facilities, both land and power, are not used immediately, losses result. Close co-ordination of the building of the project, the location and building of industry, and colonization is highly important.

The extent of integration, and the balance between agriculture and industry which is most desirable, will vary between areas. In some areas large supplies of raw products, power, and adjacent markets may make a high degree of co-ordination desirable, whereas in other areas conditions are such that little if any integration is practicable. A policy concerning the integration of agriculture and industry in irrigation areas, therefore, should be very general in nature. Whether such a plan is applicable to an individual area, and the degree of integration which is most desirable, should be determined for each proposed area after a consideration of the particular circumstances.

It is particularly to the disadvantage of Western agriculture to stress area self-sufficiency since a large part of the production of several of its most important crops depends primarily upon wide distribution. Some past attempts at area self-sufficiency made elsewhere in the United States have worked to the direct disadvantage of certain types of Western agriculture. Rather than to aim at the highest possible degree of area self-sufficiency, as the authors recommend, efforts should be concentrated upon making agricultural and manufactured products which can be produced to greatest comparative advantages in the area. For instance, parts of California have a decided advantage over most of the United States in producing oranges, but have a distinct climatic disadvantage in manufacturing cotton goods. For certain types of metal goods, any marketing advantage may be entirely outweighed by a freight disadvantage in obtaining heavy raw materials.

The authors stress the production of garden products for home use by industrial workers. In addition, some part-time farming could probably be developed to co-ordinate with industry. In present irrigation projects som farmers find periodic work in industries processing agricultural product Some co-operative plants hire mostly farmers. Successful part-time farming depends upon considerable income from sources outside the farm Outside work should be available at times not seriously conflicting wit important farm work. In addition to farmers finding work in industr through close co-ordination, industrial workers might be used on farm during periods of peak employment, such as at the harvesting of specialt crops.

Where the production of large quantities of specialty crops on ne projects is contemplated, market outlets should be carefully determine before the project is set up. A delicate balance often exists between production and consumption of such crops, which a decided increase in productio would destroy. Disturbance of the equilibrium usually causes wide pric changes. A large increase in production would probably cause great difficulties both for the new farmers as well as for other farmers already producing that crop.

Although there are large numbers of unemployed industrial workers an already over-burdened agriculture should not be expected to assimilat many of them. Rather than for the Government to encourage "back to th land" movements which tend to increase the supply and decrease effectiv demand for agricultural products, it would seem more logical to encourag existing industry, and to foster, particularly, the establishment of new industries to produce goods and services for which there would be a larg public demand. The development of new industries may offer a more important method for the solution of the unemployment problem than industrial decentralization, and appears to be worthy of much consideration

To summarize, integration of agriculture and industry should offer certain benefits. Irrigated projects often present conditions favorable to such integration. As conditions vary widely between irrigated areas, however, the adaptability of each area to such a plan should be judged individually on the basis of both economic and social values.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

DEFLECTIONS BY GEOMETRY

Discussion

By Ralph W. Stewart, M. Am. Soc. C. E.

Ralph W. Stewart,[6] M. Am. Soc. C. E. (by letter).[6a]—Adhering closely to sound conventional methods, the author has presented an able introduction to his theme and has carried the work outlined to a successful conclusion.

The only weakness which the writer is able to discover is that a part of the analysis could have been made more geometrical and less laborious. To illustrate this point the author's introductory statements regarding rotations will be slightly expanded and then the stiffness factors and other constants for the beam of variable and unsymmetrical section, represented by Fig. 10, will be developed in a more purely geometrical manner, which will substantially reduce the arithmetical work.

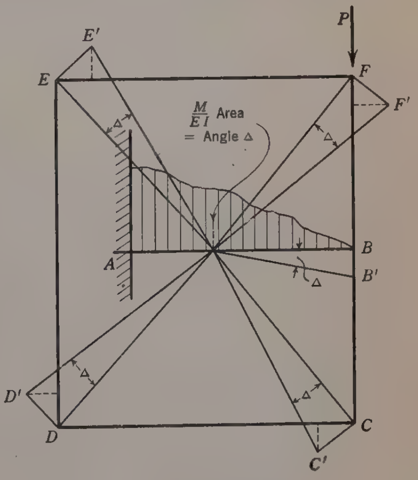

Fig. 17

Fig. 17 represents the application of the author's rotation Theorems 1 and 2, illustrated by Fig. 3, with reference to a specific cantilever beam and some connecting members, rather than with reference to an abstract point, *O*, which is not tied in to any structure. The rectangular frame, *CDEF*, is free except for its integral connection with the beam at Point *B*. It may be conceived that Sides *EF* and *DC* pass through slots in the wall which fixes the beam at Point *A*.

If now, a load, *P*, is applied either over Point *B*, or at any point on the beam, a cantilever moment will be created. Regardless as to whether the

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Note.—The paper by David B. Hall, Assoc. M. Am. Soc. C. E., was published in December, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1937, by Messrs. A. J. McGaw, and L. E. Grinter.

[6] Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

[6a] Received by the Secretary February 12, 1937.

beam section and its modulus of elasticity are constant or variable, the computer can lay out the $\frac{M}{EI}$ - diagram due to this cantilever moment, if the make-up of the beam is known. The point where the center of gravity of this $\frac{M}{EI}$ - diagram is projected upon the unsprung axis of the beam will now be the center of rotation about which the deflection of Point *B* and all points of the tributary frame occur. This center of rotation is a fixed point independent of the magnitude of the load. Its location, for any beam, can be determined by computing the $\frac{M}{EI}$ - diagram for a trial load or a trial moment when the real load or the real moment is known as to position, but not as to amount. The rotation, expressed as an angle, is equal numerically to the total area of the $\frac{M}{EI}$ - diagram.

In Fig. 17 the rotation is represented by the angle, Δ, and the movement of each lettered point on the frame is indicated by the primed letters. The formulas for the movement of each point are correctly stated by the author. A beam having two end moments will have two such centers of rotation, which, if the beam has end slopes, will be projected on the end slope lines instead of the unsprung axis.

A paper explaining the geometrical methods of analyzing the flexure of continuous structures by means of a series of lines connecting these centers of rotation, has been published.[7]

TABLE 5.—COMPUTATION OF THE CONSTANT FOR THE BEAM IN FIG. 10

Point * (1)	Moment, M (2)	Values of $\frac{M}{t^3}$ (3)	Moment of Column (3) (4)	Point * (1)	Moment, M (2)	Values of $\frac{M}{t^3}$ (3)	Moment of Column (3) (4)	Point * (1)	Moment, M (2)	Values of $\frac{M}{t^3}$ (3)	Moment of Column (3) (4)
A	13	0.124	0.788†	5	8	2.856	22.848	10	3	0.302	0.906
1	12	0.228	2.736	6	7	3.668	25.676	11	2	0.095	0.190
2	11	0.460	5.060	7	6	2.928	17.568	12	1	0.022	0.022
3	10	0.903	9.030	8	5	1.900	9.500	*B*	0	0	0
4	9	1.678	15.102	9	4	0.844	3.376				
								Summation....		15.945	112.802

* See Fig. 10. † $\frac{0.124}{2} \times 12\frac{2}{3} = 0.788$.

In an analysis such as that of the variable beam (Fig. 10), it is better to find these centers of rotation and the ratio between an end moment and its corresponding angle of rotation and subsequently to treat the flexure in the span due to each end moment as a single item rather than as a series of small items each resulting from the curvature of a separate increment, *dl*. This permits the mechanics of the problem to be transformed into a purer form of geometry and also reduces the number of arithmetical operations.

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[7] "The Analysis of Continuous Structures by Traversing the Elastic Curves", by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105.

To compute the constant for the beam in Fig. 10 apply a trial end moment of 13 at End A with End B assumed to be hinged. In Table 5, Columns (1) represent the moment at each division point of the beam and Columns (2), the $\frac{M}{t^3}$ - value at each division point. The $\frac{M}{t^3}$ - area, taking each division length as unity instead of 3.52, is the summation of Columns (2), except that before adding, the top and bottom items are divided by 2 as they apply to only one-half as much length as the others.

To get the position of the center of gravity of the $\frac{M}{t^3}$ - area the moment of each elementary area about Point B is computed as shown in Columns (3) still taking the dl lengths as unity instead of their real lengths.

The relative distance to the center of gravity of the $\frac{M}{t^3}$ - area is now the summation, Columns (3) divided by the summation of Columns (2); thus (see Table 5): $\frac{112.802}{15.945} = 7.06$. If this actual distance is desired, multiply the relative distance by 3.52, the division length. A similar computation is made for a trial moment of 13 at Point B with Point A hinged, or by utilizing Maxwell's theorem of reciprocal deflections, the computation for the center of rotation for a moment at B can be made with one less column.

It is found that a trial-end moment of 13 at Point A gives an $\frac{M}{t^3}$ - area of 15.94 with its center of gravity 7.08 division lengths from Point B, and a trial-end moment of 13 at Point B gives an $\frac{M}{t^3}$ - area of 16.27 with its center of gravity 7.19 division lengths from Point A.

All constants for the beam can now be taken by simple ratio and proportion from the geometry of the traverse diagrams, Fig. 18. These diagrams are similar to the author's Fig 10 (a), except that they are completed to show the centers of rotation which correspond to Point O in Fig. 3, or Δ in Fig. 17. Fig. 18 shows a rotation of unity at Point A.

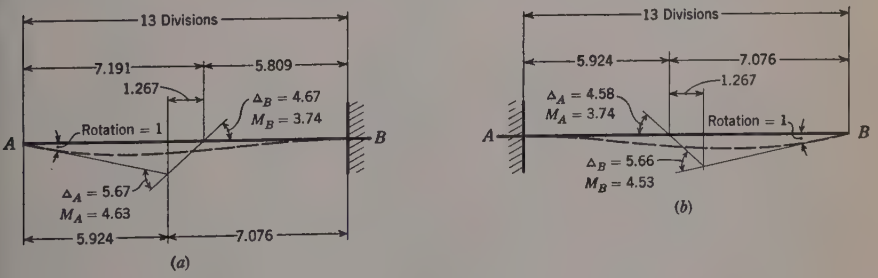

FIG. 18

The other angles are evaluated from the fact that in the traverse triangle the angles are proportional to the opposite sides. The moments appurtenant to the two Δ -angles bear the ratio to the trial moment of 13 that their Δ -angles bear to the $\frac{M}{t^3}$-area for the trial moment of 13. It is now seen at once that a unit rotation at Point A with Point B fixed gives $M_A = 3.74$, which is the stiffness factor as defined by the author; and a unit rotation at Point B with Point A fixed gives $M_B = 4.53$. These agree with the stiffness factors computed by the author (see, following Table 3). The carry-over factors are also taken off these traverse diagrams by dividing the moment at the fixed end by that at the rotated end.

The saving in computation as compared with Table 2 is more than appears at first glance, because when a slide-rule is set for an $\frac{M}{t^3}$-value in Table 5[8] it is only necessary to move the runner to the square scale, to extend the computation to Columns (3), since the selection of 13 for the trial moment makes $\frac{M\,x}{t^3} = \frac{M^2}{t^3}$. Taking stiffness and carry-over factors from these traverse diagrams is recommended as being superior to the algebraic solution by means of the Equations (7) to (16), inclusive.

At the end of the paper the author suggests some exercises for any one wishing to acquire proficiency in the geometric treatment of deflections. The last exercise suggested is to devise a method of computing the dead load moments in a two-hinged arch and then find what the dead load moments will be if the structure is transformed into a fixed arch by eliminating the rotation at the base of the two-hinged arch. This problem can be made short and its solution conveniently given in full by selecting, for the arch, a portal frame with level deck and vertical abutments, with uniform moment of inertia throughout, and height = length = L.

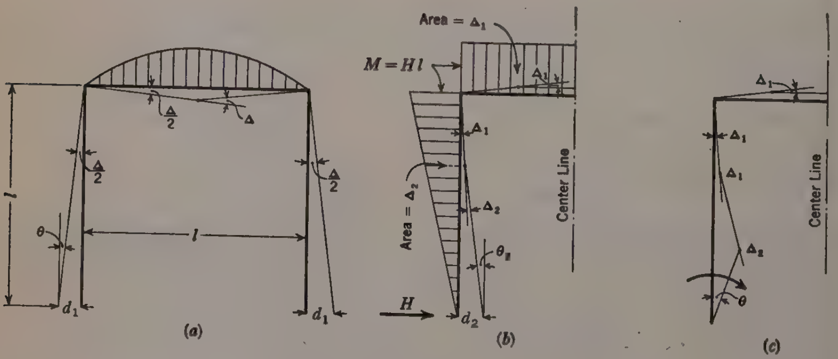

FIG. 19.

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[8] A complete explanation of the technique of Table 5 may be found in *Bulletin No. 66*, Ohio State Univ., Columbus, Ohio.

In the following solution the Δ-angle points represent centers of geometrical rotation which result from appurtenant sections of the $\frac{M}{EI}$ - diagram. Since E and I (or t^3) are constant throughout, they may be omitted from the computations. For Condition No. 1 (Fig. 19 (*a*)), the arch is taken as a simple structure with no restraint, subjected to a uniform load, w, per unit of length.

The moments are: At the crown, $\frac{1}{8}\, wL^2$; at the haunch, 0; and at the column base, 0. The area of the moment diagram equals,

$$\Sigma M \; dL = \frac{1}{12}\, w\, L^3 = \Delta \; \ldots\ldots\ldots\ldots\ldots\ldots \text{(30)}$$

The spread at the bottom of the columns equals,

$$2\, d_1 = \Sigma M \; dL \; y = \Delta\, L = \frac{1}{12}\, w\, L^4 \; \ldots\ldots\ldots\ldots\ldots \text{(31)}$$

and,

$$\theta = \frac{\Delta}{2} = \frac{1}{24}\, w\, L^3 \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(32)}$$

For Condition No. 2 (Fig. 19(*b*)), the structure is converted into a two-hinged arch by applying a thrust, H, sufficient to bring the column bases to their original positions: $d = \frac{1}{24}\, w\, L^4 = \Sigma\, M \; dL \; y = \Delta_1\, L + \Delta_2 \left(\frac{2}{3}\, L\right) = \frac{H\, L^3}{3} + \frac{H\, L^3}{3} = \frac{5}{6}\, H\, L^3$. Therefore,

$$H = \frac{1}{20}\, w\, L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(33)}$$

The moments in the structure now are the combination of the moment in Fig. 19(*a*) with the moments in Fig. 19(*b*), as follows: At the crown, $\frac{1}{8}\, w\, L^2 - \frac{1}{20}\, w\, L^2 = \frac{3}{40}\, w\, L^2$; at the haunch, $\frac{1}{20}\, w\, L^2$; and, at the column base, 0. The rotation at the column base due to the moments shown on Fig. 19(*b*) is: $-\,\Delta_1 - \Delta_2 = -\,\frac{1}{20}\, w\, L^3$. Combining this with the rotation of $+\,\frac{1}{24}\, w\, L^3$ in Fig. 19(*a*), the resultant rotation is $-\,\frac{1}{120}\, w\, L^3$. To convert the arch into a fixed-base arch, the column base (Fig. 19(*c*)) is made vertical by rotating it through the angle, $\theta = \frac{1}{120}\, w\, L^3$, in a clockwise direction and the resulting moments are determined and combined with those of Fig. 19(*a*) and Fig. 19(*b*). To solve Fig. 19(*c*), a traverse computation is made from the top to the bottom of the column, calling clockwise rotations positive: $-3\Delta_1 - 2\Delta_1 + \Delta_2 = 0$. Therefore, $\Delta_2 = 5\Delta_1$. Summing the angles: $3\Delta_1 = \frac{1}{120} w L^3$; M (at the haunch) $= \frac{2\Delta_1}{L} = \frac{1}{180}$ $w L^2$; M_2 (at the column base) $= 5 M_1 = \frac{1}{36} w L^2$; and the crown moment $=$ the haunch moment $= \frac{1}{180} w L^2$.

Combining these quantities with the moments in Fig. 19(*a*) and Fig. 19(*b*), the respective moments for the fixed arch are as follows: Crown moment, $\frac{1}{8} w L^2 - \frac{1}{20} w L^2 - \frac{1}{180} w L^2 = \frac{5}{72} w L^2$; haunch moment, $\frac{1}{20} w L^2$ $+ \frac{1}{180} w L^2 = \frac{1}{18} w L^2$; and, moment at the column base, $\frac{5}{180} w L^2$.

The foregoing analysis carries the structure progressively, by a geometric treatment of its deflections, through the three conditions of free base, hinged base, and fixed base.

Except for the parts which fail to utilize fully the geometrical properties of flexure (notably the part relating to the constants for a variable section beam), the paper merits attention.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ECONOMIC DIAMETER OF STEEL PENSTOCKS

Discussion

BY ADOLPHO SANTOS, JR., ASSOC. M. AM. SOC. C. E.

ADOLPHO SANTOS, JR.,[22] ASSOC. M. AM. SOC. C. E. (by letter).[22a]—The subject of this paper belongs to that class of engineering problems which, for their proper solution, require the exercise of judgment. Affected as they are by matters of opinion and by a multiplicity of varying, complex, and, sometimes, conflicting factors, such problems cannot be solved with any high degree of precision no matter how dexterous one may be in the use of mathematical tools. Attempts to increase the precision beyond the limits imposed by the nature of the problem are illusory and only add to the complexity. The principal merit of mathematical analyses of such problems is as an aid in understanding them and in reaching sound judgment.

The authors state that, as computed by their formulas, the economic diameter of a penstock for a certain development shows a difference of only 3.25%, as compared to the diameter determined from detailed studies. To the writer this only proves that, starting from identical, basic, and assumed data, the formulas give results which agree with those obtained from detailed studies. It does not prove that the assumptions were correct. Certain of these assumptions, and the haze of uncertainty surrounding them, will be discussed briefly.

The selection of Scobey's formula for estimating the penstock friction loss is justified. It is indeed authoritative, but any estimate of friction-head loss based on it, intended to serve as the basis of economic studies, is nothing more than a forecast of future conditions and, as such, it is not necessarily accurate. The coefficient, K_s, depends on the class of pipes and joints but varies considerably for different penstocks of the same class, and, in many cases, the class itself is only definitely known after the final award of the contract, as joints and other details differ between different manufacturers. It is also known that K_s increases with the age of the penstock,

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NOTE.—The paper by the late Charles Voetsch, M. Am. Soc. C. E., and M. H. Fresen, Assoc. M. Am. Soc. C. E., was published in November, 1936, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: March, 1937, by Messrs. R. A. Monroe, William E. Rudolph, and Peter Bier.

[22] Hydr. Engr., The São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

[22a] Received by the Secretary February 19, 1937.

the rate of increase depending on the size of the penstock, the type o protective cover of the internal surface, the quality of the water, climate, etc. The writer knows of cases in which the rate of increase of K_s is 10% each year, and of an extreme case in which it is more than 20% per year. The question arises, therefore, as to what rate of increase and what age of the penstock should be assumed for purposes of economic studies.

The assumption that the friction loss in the penstock varies as the 1.9 power while the other hydraulic losses vary as the square of the velocity seems to be an unwarranted refinement in economic studies because: (*a*) K_s cannot be predicted accurately; (*b*) "* * * it must not be overlooked that the individual values of z vary all the way from 1.73 to 2.14", as Scobey has stated;[23] (*c*) the refinement adds greatly to the complexity of the problem, as will be seen by comparing Equation (11) with Equations (24) to (27); and (*d*), the gain in precision, if any, is negligible.

It would be preferable to select a value of K_s high enough to cover the total losses, particularly in the more common developments where the bend losses are only a small proportion of the total. In the relatively few cases where the bend losses form a great proportion of the total it might be better, in order to avoid unnecessary complexity, to adopt some formula other than that of Scobey in which the friction loss is made to vary as the square of the velocity.

There is also the question of the discharge which, as the authors state, "should be estimated as closely as possible". For the purpose of economic studies, should the maximum overload, the rated, the most frequent, or some other value of discharge, be assumed? This question is answered partly by the introduction of a factor to take care of the effect of the load factor. The question is not precisely answered, however, because to apply this factor it is necessary to make assumptions regarding the future loading of the unit, and this, again, requires the introduction of the element of judgment. In any case, no matter what value of discharge is selected, the fact remains that the horse-power and efficiency guaranties given by the turbine manufacturers are subject to a certain tolerance, so that at the time of making the economic studies the discharge is not known with a precision greater than 1 to 2 per cent.

In order to determine the thickness of the steel pipes it is necessary to assume a value for the head. This is not quite as simple as it appears. For reasons given elsewhere[24] it is considered that high-head penstocks, at least, should be designed to resist, without appreciable deformation, the stresses caused by the extreme pressures resulting from accidental conditions. These extreme pressures may arise from a variety of causes, some of which depend on the maximum discharge whereas some do not. The probability of any one assumed accidental condition arising during the life of the penstock is a matter for speculation so that weighting is difficult. Then there are the questions of maximum allowable stress intensity under static pressure in the lower part of the penstock, and of minimum thickness to prevent collapse due to vacuum, and as protection against corrosion, in the upper part. As a result of these and other factors it is a nice problem for the judgment and experience of the engineer to determine just which head to use.

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[23] "The Flow of Water in Riveted Steel and Analogous Pipes", by Fred C. Scobey, M. Am. Soc. C. E., *Technical Bulletin No. 50*, U. S. Dept. of Agriculture, January, 1930, p. 81.

[24] "High-Head Penstock Design", by Messrs. Billings, Dodkin, Knapp, and Santos; Symposium on Water-Hammer, A. S. M. E., Hydraulic Div., and Am. Soc. C. E., Power Div., p. 29 *et seq.*

All other factors entering into the problem, such as, over-all efficiency of the plant; loss factor corresponding to the load factor under which a plant operates (what should one assume as the period of the average load and as the interval of the maximum load?); value of power lost; unit cost of steel (which varies with the diameter of the penstock not only in purchase price, but also as affected by erection costs); percentage of over-weight; etc., are, each and all, subject to varying degrees of uncertainty. This being the case it follows that the calculated economic diameter is also uncertain and can serve only as a guide to judgment. The probable error or degree of uncertainty of the computed value can be estimated by the well known law of the propagation of errors.

For purposes of illustration the writer considers the data given by the authors and assumes certain values for the probable errors of some of the factors: $Q = 400 \pm 4$ cu ft per sec; $e = 0.75 \pm 0.015$; $K_s = 0.401 \pm 0.020$; $K_e = 0.05 \pm 0.005$; $k_b = 0.25 \pm 0.025$; $H = 130 \pm 10$ ft; $i = 0.20 \pm 0.05$; and, $f = 0.25 \pm 0.025$. It will be assumed that the remaining factors: Δ, L_p, L_f, a, r, s_g, e_j, and b are known precisely (which is not the case), and are as given by the authors.

Applying to these data, the law of the propagation of errors, the writer finds from Equation (2) that the head lost due to friction is 0.421 ± 0.022 ft. Similarly, from Equations (2) and (16), it is found that the total head loss is 0.680 ± 0.030 ft. In other words, the probable error of the friction loss is about $\pm 5.2\%$ and that of the total loss, $\pm 4.4\%$, thus indicating that the two methods of arriving at the head loss are of about the same degree of precision. The reason for the large difference in the actual loss as computed by the two methods lies in the fact that the bend losses form a high proportion of the total. This difference could have been made to disappear by increasing K_s proportionately.

Keeping the same number of significant places as the authors, it is found, on the basis of Equations (23) and (26), that $G = 1\,058 \times 10^7 \pm 61 \times 10^7$; $M = 88.3 \times 10^7 \pm 7.3 \times 10^7$; $J = 13.14 \times 10^{-4} \pm 1.34 \times 10^{-4}$; $B = 16.4 \pm 1.4$; and $D = 7.315 \pm 0.154$ ft. This shows that the probable error of the computed value of the economic diameter is about $\pm$ 2 in., even when it is optimistically assumed that one-half the factors affecting the result are known exactly. If the retardation coefficient is increased by 15% (and there is reason for doing this when the bend losses are known to be high), thus making $K_s = 0.461$, the diameter computed by the simpler Equation (11) is 7.10 ft, with about the same probable error as the value previously determined.

While on the subject of precision of computations the writer would remark that in deducing Equation (4) from Equation (3), there is no justification for increasing the number of significant places in the constants from three (62.5 and 0.746) to six (1.17631). Similarly, in going from Equation (16) to Equation (17) the increase in the number of significant figures from four to seven is not justified. It is an old rule that "one can never acquire in the product more accurate places than there are in the poorer of the two factors". Division comes under the same rule.

The writer has tried to emphasize the limitations of all formulas purporting to give, directly, the economic diameter of steel penstocks. A further limitation of any such formulas results from the fact that they give one, and only one, value for the diameter, whereas, at least in the case of high-head penstocks, it will be found more economical to vary the diameter with the head. In such cases, it will also be found expedient to proportion the diameters and the length of each reach of different diameter, aside from economical considerations, in such a manner as to reduce to a minimum the pressures resulting from the water-hammers produced during accidents.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

FILTER SAND FOR WATER PURIFICATION PLANTS

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON FILTERING MATERIALS FOR WATER AND SEWAGE WORKS

Discussion

BY MESSRS. J. W. ELLMS, THOMAS R. CAMP, WILLARD C. LAWRENCE, G. F. GILKISON, L. F. ALLAN, AND O. J. RIPPLE

J. W. ELLMS,[11] M. AM. SOC. C. E. (by letter).[11a]—The work done by this Committee has produced some valuable information. The studies indicate that the problem is somewhat more complicated than it would first appear. The ideal filtering medium should provide a complete removal of suspended impurities. It should also be of such a character that it can be perfectly cleaned by the washing process, since a failure in this respect renders the medium eventually unfit for further use. It is on this phase of the problem that the writer wishes to place especial emphasis.

Since sand is the filtering medium that is most commonly used, it is the material which must be given the greatest consideration. It is the medium that is usually available and that may be obtained at the lowest cost, both of which conditions will largely control in any filtration project. If a uniform size of sand grain seems to produce a better removal of impurities and makes possible more effective washing, then almost any natural sand will require considerable preparation in the form of grading and washing. In some cases this preparation may increase the cost materially.

The coarser the sand the farther the suspended impurities will penetrate the sand bed. In consequence, there must be some optimum depth of bed for each size of sand grain, if some limiting depth of penetration is to be maintained. A sand that is so fine as to produce a compacted mat at the surface which is not easily broken up by washing, is obviously undesirable; on

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NOTE.—This report was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1936, and published in the December, 1936, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[11] San. Engr., Dept. of Public Utilities, Cleveland, Ohio.

[11a] Received by the Secretary February 1, 1937.

the other hand, a sand that is so coarse that the entire bed is permeated by the suspended impurities, will require a high rate of wash-water flow in order to flush the entrapped impurities thoroughly.

There are certain other secondary conditions produced by the choice of a filter sand, that affect both the efficiency of filtration and the cost of equipment. The shape of the sand particle, and its roughness, may influence the extent to which the impurities adhere to the sand although this factor may be of minor importance. Poor adhesion of the suspended impurities would mean a greater degree of penetration, whereas if the sand held the entrapped material too tenaciously, washing it off would become more difficult. Expansion of the sand bed, if high velocity washing is practiced, necessitates increased wash-water velocities for the coarser sand. With increased velocities for the wash water, larger valves and underdrain piping will be needed and will thus add to the cost of construction.

Whether they are constructed of sand grains of uniform size, or sand hydraulically graded from coarse to fine, sand filter beds must have a properly constructed supporting layer. If gravel is used for this supporting layer, it will have to be graded so that the sand will not penetrate too far into it. Theoretically, sand should not penetrate into such a layer at all, but in practice it is usually found to do so to a greater or less degree.

Finally, the type of underdrain system used may have a very important effect upon the efficiency of the most perfectly designed sand filter bed. Imperfect washing which permits accumulation of impurities within the bed, may require many years to develop. Eventually, these cumulative effects produce a sand bed loaded with organic matter mingled with inorganic impurities which have not been washed out of the bed. Bacteria multiply in such a bed, especially during the months when the water passing through is at a higher temperature, and decomposing organic matter may impart an odor, or even a taste, to the filtered water.

Considering all the numerous factors involved in providing an ideal filtering medium, and further noting that conflicting conditions may result by requiring certain properties in the sand used in the bed, it seems to the writer that a compromise will have to be made by balancing the more desirable characteristics of the sand against those that are less desirable in order to obtain a practical design.

It is understood that the section of the report entitled "Basic Considerations and Definitions Applying to the Experiments", is for convenience in discussing the data obtained. It would be unfortunate, however, if all these definitions and conditions were to be accepted as ideal. No sand filter bed ever goes into service entirely clean; rates of filtration greater or less than 125 000 000 gal per acre per day are entirely satisfactory in some cases; bacterial purity has some, but not a very definite, relation to the clarity of the filtered water; and critical depth is a term used in hydraulics, having a very definite meaning quite foreign to the sense in which it is utilized in this report. However, the ingenuity shown by the Committee in devising methods and apparatus for a critical analysis of the complex conditions involved in sand filtration problems deserves commendation.

THOMAS R. CAMP,[12] M. AM. SOC. C. E. (by letter).[12a]—The Committee has done a splendid piece of work in its study of filter sands. As pointed out in the progress report, any consideration of filtering materials should be related to the other factors that influence filter performance and costs if the studies are to be of ultimate use in improving plant design. These factors include, in addition to the sand itself: The rate of filtration; the viscosity of the water; the number and size of suspended particles in the water; the depth of the bed; the method and efficiency of washing; and all costs both capital and operating. After nearly ten years of study by the Committee many of these factors have scarcely been touched upon, and the relations between them are only beginning to become apparent. Such is the complexity of the problem. Nevertheless, real progress has been made, as can readily be appreciated by any one who has attempted experimental work in filtration. The work of the Committee should continue.

If the writer may presume to state for the Committee the ultimate object of the studies, he would say that it is to learn enough about the process of filtration so that filter plants may be designed to produce a water of desired quality at all times, with a minimum of cost. In the past, filters have been designed with 24 to 30 in. of sand, and to operate at a rate of about 2 gal per min per sq ft. In general, the results have been satisfactory as far as water quality is concerned. That is not to say, however, that such designs are economical. If the rate of filtration can be increased manyfold without too great an increase in depth or other costs, economies will result. Rates as high as 3 gal per min per sq ft are now (1937) being tried without substantial modifications in plant design. There is reason to suppose that much higher rates can be used if the proper modification of other factors influencing performance is effected.

A large number of experimental studies dealing with filtration were made under the writer's direction at the Scituate Purification Works of the City of Providence, R. I., during 1934 and 1935. The experimental filter had a surface area of 2 sq ft. During the period from January 31, to July 10, 1934, 33 runs were made with this unit.[13] Further runs were made by Dr. Rolf Eliassen[14] during 1934 and 1935 for the express purpose of studying the theory of filtration.

Two major differences in the technique used in these runs, as compared to that used in the experiments performed under the direction of the Committee, are worthy of comment. Piezometer connections were provided through the side wall of the filter at intervals of 3 to 6 in. in height. Head loss was thus measured through five or more parts of the filter depth as well as through the filter as a whole. Moreover, water samples were withdrawn through these connections for analysis during the run. The results obtained per run were thus increased manyfold over the number of data possible without such side connections.

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[12] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[12a] Received by the Secretary February 4, 1937.

[13] "Experimental Study of Porous Plates for Use in Filter Bottoms for Rapid Filters", by Thomas R. Camp, *Journal*, New England Water Works Assoc., March, 1935, p. 1.

[14] "An Experimental and Theoretical Investigation of the Clogging of a Rapid Sand Filter", presented as a thesis to Mass. Inst. Tech., in 1935, in partial fulfillment of the requirements for the degree of Doctor of Science.

The other major difference was in the use of the iron determination as a measure of the quantity of suspended matter in the water, in preference to a turbidity test. This choice is not possible where alum is used as the coagulant, but was possible at Providence because ferric sulfate was then being used as the coagulant. The turbidity test, as emphasized by the Committee, is a very poor measure of the suspended matter in the water. Tests for suspended solids are very difficult to make because of the small quantities of suspended matter present in water. Iron tests, on the other hand, can be made accurately. Investigations at Providence indicated that substantially all the iron was in solid form, either suspended or colloidal. Tests by Eliassen indicated that the ratio of iron to suspended solids in the water applied to the filter was substantially constant.

Factors Affecting Filter Performance.—The influence of some of the factors affecting filter performance may be estimated by a study of theory already developed in conjunction with results of recent experiments. The flow of clean water through clean sand is represented accurately by the Fair-Hatch[15] equation, which includes all the factors known to influence the flow and which also appears to be rational. The Fair-Hatch equation in its simplest form is as follows:

$$\frac{h}{d} = \frac{k\,\mu}{w}\,V\,\frac{(1-P)^2}{P^3}\,\frac{s^2}{D^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which (in the notation of the report) h = the lost head through the depth, d; k = a factor having a value of about 4, but varying with the degree of compactness of the bed; μ = the coefficient of viscosity; w = weight of water per unit volume; V = the rate of filtration expressed as a velocity; P = the porosity of the sand; s = the shape factor of the sand grains; and, D = the size of the sand grains.

Equation (4) assumes that the porosity is uniform throughout the bed and that all the sand grains have the same shape and size. The value of the factor, k, is influenced by the manner of expressing the grain size. For a filter sand, which is never perfectly uniform in size and which becomes stratified after washing, D must be expressed as a function of d and the expression must be integrated over the filter depth to obtain the total head loss. Moreover, it is quite likely that both s and P, and possibly k, will vary somewhat with depth and should be treated likewise.

Equation (4) may be applied quite as well to a filter bed during filtration as to clean sand and clean water if it is remembered that it expresses the momentary relation between the variables. The quantities also vary with time, however, due to the clogging of the filter. Thus, it is well known that the porosity, P, decreases with clogging. It is also probable that both shape factor and size of grains change with clogging, because the filter medium consists, during a run, of the deposits within the bed as well as the sand. Just how these variables change with clogging remains to be determined.

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[15] "Fundamental Factors Governing the Streamline Flow of Water Through Sand", *Journal,* Am. Water Works Assoc., November, 1933, p. 1551.

The time rate of clogging depends upon the quantity of suspended matter and the size of the suspended particles in the applied water, and possibly upon other factors in addition to those given in Equation (4). The depth of penetration into the bed also depends upon the size of the suspended particles. Hence, at least three more variables must be added to the seven given in the equation; as follows: (*a*) The time or length of run; (*b*) the quantity of suspended matter; and, (*c*) the size of the suspended particles. The quantity and size of the suspended particles are functions of the quality of the raw water and the pre-treatment—that is, of the coagulation and settling processes.

In Equation (4), the effect of temperature is exhibited in the viscosity, and the relation between the two variables is well known. The temperature exhibits a different effect upon the quantity and size of suspended particles through its influence on coagulation and sedimentation; and, therefore, in the filtration process its effect is felt twice.

Porosity.—The porosity of the bed is the most important single factor in the filtration process. Even in a clean sand bed, its effect is marked. For example, from Equation (4), a change in porosity from 0.41 to 0.40, or a corresponding 2.5% error in its determination, changes the lost head by nearly 10 per cent. According to estimates of Eliassen, the porosity in the top layer of sand may be reduced to less than 0.05 by clogging. A change of porosity of one point from 0.06 to 0.05 increases the lost head 75 per cent. The importance of the accurate determination of porosity in experimental work is thus apparent.

The average porosity in a bed of clean sand is quite easily measured. It must be determined, however, with the sand in position ready for filtration. There is no single value of porosity corresponding with a sand of given size and shape. A 24-in. nominal sand depth may vary by a half inch or more, when the sand settles back after washing, depending largely upon the rate of closing the wash-water valve. A variation of a half inch in depth changes the value of the average porosity by about 5 per cent.

The average porosity is of little value for filtration studies. Even in clean sand the porosity is probably greater at the top of the bed than below. In a filter bed, immediately after washing, the sand grains at the top do not pack as tightly as the larger grains below; and, moreover, they are held farther apart by the floc which they have retained after washing. During a run the porosity at the top becomes much less than that below, due to clogging, and also to shrinkage in the case of fine sands. No satisfactory means has yet been devised for measuring the porosity at any depth in a bed. It is this value which is needed for the application of the theory. Any means devised for measuring the porosity should be capable of obtaining it at any depth and at any time during a run, without disturbing the filtration process.

Sand Size.—The writer agrees with Mr. Armstrong in his belief that the sand size can be revealed adequately only by a complete sieve analysis (see heading "Designation of Sand Size"), and that the top size is more indicative of the character of the sand in rapid filters than the Hazen "effective size." It is of little importance whether the size of grain is determined from the size of the sieve opening or by the "count-and-weigh" method of Hazen, provided uniformity of results can be obtained. In making a sieve analysis it is important to have a large number of sieves. The number of sieves now available in the complete Tyler standard scale is insufficient to describe a uniform filter sand adequately.

The size of grain is not sufficient to describe the character of a filter medium. A shape factor and the specific gravity are also important, the latter affecting particularly the effectiveness of washing.

Washing Filters.—The method recommended for determining the efficiency of washing is a good one (see "Part II—Washing Filters"). The writer suggests, however, that it might be improved by using the suspended solids determination instead of the turbidity test.

It is to be regretted that uniform methods of washing were not adopted by the plants co-operating in the experimental work. More regularity in the results could have been obtained if the same rate of wash and length of wash had been used by all. The rate of wash for each filter should be sufficiently high to suspend all the sand at the warmest temperatures. The rate and time of opening and closing of the wash-water valve are also important.

The cleansing action is a twofold process consisting of the separation of the floc from the sand grains and the transportation of the floc upward and out of the filter. The separation process depends for its effectiveness upon the magnitude of the friction between the water and the sand grain. The rubbing together of sand grains is probably of minor importance. The magnitude of the friction force is equal to the weight of the sand grain in water; so, also, is the total head loss through the sand during washing equal to the head corresponding to the weight of all the sand in the water. The significance of this relation does not seem to be widely appreciated.

Fig. 17 shows the data for one of the washes during the experiments at Providence, R. I. The total head loss checked accurately with the weight of the sand in water. If the loss is less than the sand weight, some of the sand is not in suspension. An increase in the expansion of the bed after all the sand is in suspension has no effect upon the magnitude of the friction loss, and, hence, it has no effect upon the effectiveness of separation of the floc from the sand grains.

The rate at which the floc is transported upward above the sand at a given wash rate is greatest in winter due to the greater viscosity of the water. The velocity of transportation through the sand, however, is probably greatest in summer because the sand is not expanded so much, and, therefore, the water velocity through the pores is higher.

Rate of Clogging of Filters.—The most convenient measure of the clogging rate is the rate at which the head loss builds up. In experiments at Detroit, Mich., by Messrs. Hulbert and Herring[16], from 1927 to 1929, it was found that the loss of head increased at practically a constant rate when the water quality and rate of filtration were constant. The writer was intrigued by this finding, and made inquiries to determine if the same condition existed elsewhere. A study of many runs at the Cambridge, Mass., filter plant indicated the same tendency. Many of the experimental runs at Providence in the spring of 1934 indicated a straight-line rate of increase of lost head,

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[16] "Studies on the Washing of Rapid Filters", *Journal*, Am. Water Works Assoc., November, 1929, p. 1445.

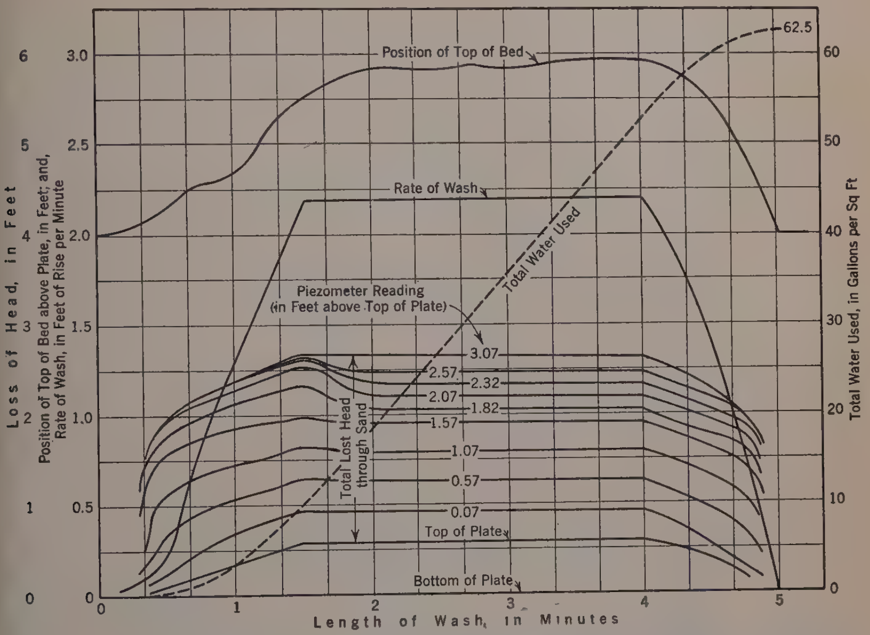

FIG. 17.—DATA FOR WASH NO. 7 (5.4% OF WASH WATER FILTERED; WATER TEMPERATURE 34°.5 F. AND CAPE COD SAND, 23.75 INCHES DEEP; EFFECTIVE SIZE, 0.48 MILLIMETER; AND UNIFORMITY COEFFICIENT = 1.33).

although many other runs showed an increasing rate with time. Fig. 18 shows the rate at which the head loss built up in one of the runs made by Eliassen, most of which runs show a rate increasing with time. If a rational explanation of the shape of the head-loss curve can be derived, it will aid materially toward the development of a theory of clogging.

In the 1934 experiments at Providence, facilities were available for applying any desired mixture of settled and unsettled water to the filter. Inasmuch as a removal of approximately 90% was obtained in the sedimentation basins, a quite variable quality of water could be obtained in the mixture. The experiments indicated roughly that the rate of clogging is directly proportional to the rate of filtration and to the concentration of suspended matter in the applied water, other factors remaining unchanged. With the same quantity of suspended matter, a reduction in the size of suspended particles will result in greater penetration into the bed and, therefore, in increased length of runs. As sedimentation reduces both the quantity of suspended matter and the size of the particles, effective settling increases filter runs enormously.

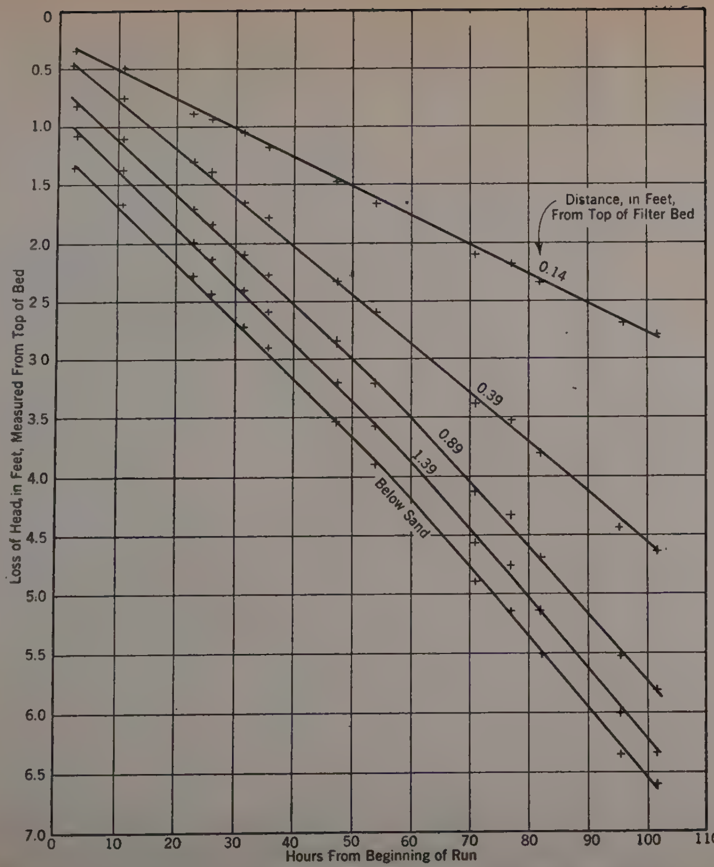

FIG. 18.—LOSS OF HEAD *vs.* TIME AT VARIOUS DEPTHS IN

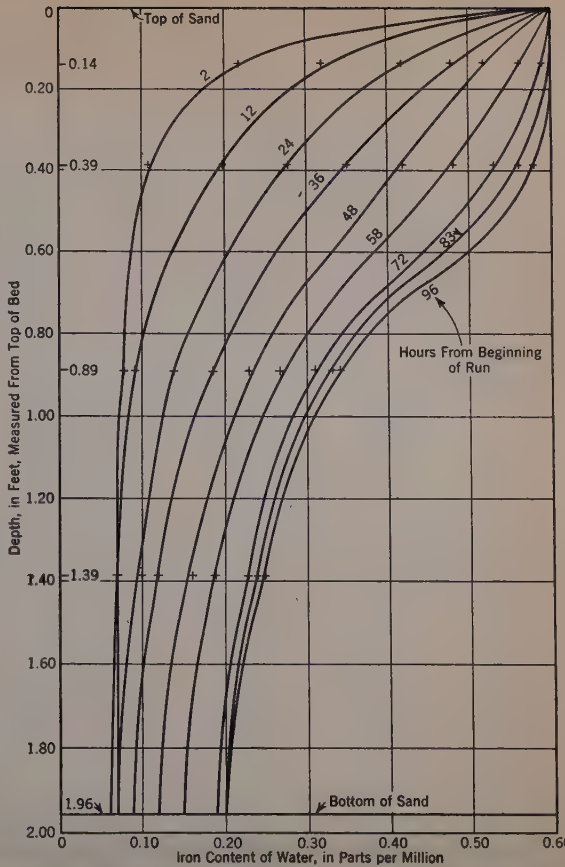

FIG. 19.—IRON CONTENT *vs.* DEPTH IN FILTER AT

The data in the report, on length of filter runs, indicate increased length of run with increase in sand size. This is to be expected as a greater depth of penetration can be secured by either a decrease in the size of the suspended particles or an increase in the size of the pores through which they must pass. The variation in the results is doubtless due to the variation among the runs of the quantity and size of suspended matter as well as the temperature. Only the temperature was reported.

Penetration of Suspended Matter.—The use of the iron test for suspended matter at Providence made it possible to observe penetration more readily and more accurately than is possible by means of turbidity tests. Fig. 19 shows the penetration of iron into the bed as determined by Eliassen during the run the head loss curves of which are illustrated in Fig. 18.

The poor removal obtained in this run is explained by the fact that the top sand size was made coarse enough to insure penetration of all the floc into the bed. In regular plant operation most of the floc is deposited near the top of the bed, and some of it settles upon the top of the sand. Removal is thus accomplished by both filtration and sedimentation. In order to study filtration, it is convenient to eliminate the other process. Ottawa sand, graded in accordance with Fig. 20, was used in this run, and the rate of filtration was 2 gal per min per sq ft. The average porosity of the clean bed was 0.408.

It will be noted by studying Fig. 19 that the rate of removal was continuously decreasing in the top layer of the bed and increasing below. This transfer of the burden of removal from the upper to the lower part of the bed is illustrated clearly in Fig. 21. After about 80 hr no measurable amount of removal was occurring in the top filter sand. Nevertheless, as may be noted from Fig. 18, the slope of the head-loss curve continued uniformly. This seemingly paradoxial result may be explained by the fact that removal is still occurring in the top layer, but in very small quantities. Since the porosity has been reduced so much, the effect on the head loss of very small quantities of deposited matter is great.

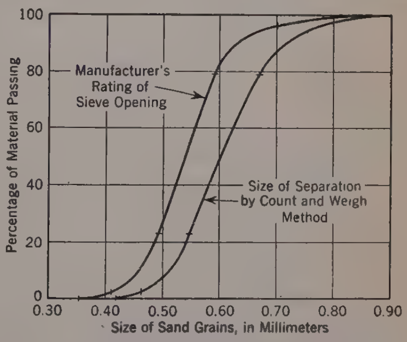

FIG. 20—SIEVE ANALYSES OF SAND IN FILTERS.

Size of Floc Particles.—In the routine operation of the Providence filter plant about 0.06 ppm of iron appeared in the effluent. The corresponding color was less than 10 ppm and the turbidity less than 1. Since the iron content of the treated water was about 4 ppm during the spring of 1934, the corresponding removal in the settling tanks was about 90%, and the removal in the filters nearly as much again. All the iron removed was doubtless in suspension. The residual of about 1% in the filter effluent was also largely in suspension, although the particles could not be seen under a 100-power microscope without dyeing them. In the experiments with porous plates for the filter bottom, this small residual of iron in the filter effluent was responsible for the clogging of the plates. About 20% of the residual in the water was removed by the plates.

Observations were made by Eliassen of the floc particles in the water samples taken from various points in the bed. A 100-power microscope

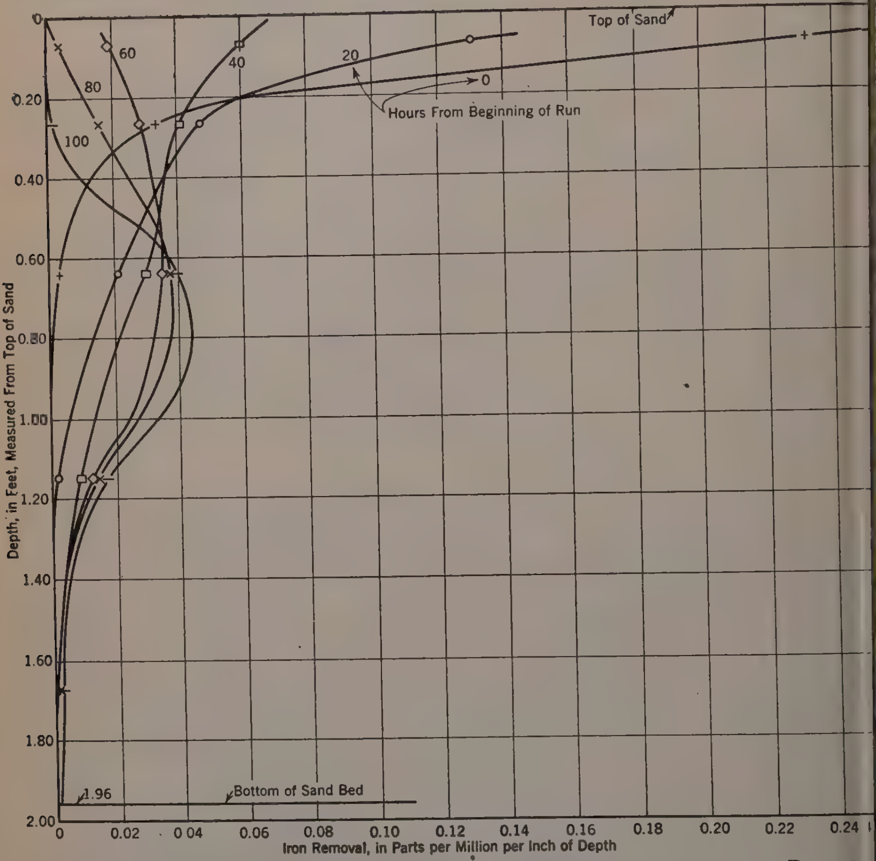

FIG. 21.—IRON REMOVAL PER INCH OF DEPTH *vs.* DEPTH AT VARIOUS TIMES DURING THE RUN.

equipped with an ocular micrometer, was used. The floc particles were s transparent that they could not be seen until they were dyed a deep red wit carbol fuchsin stain. In the settled water applied to the filter the concentration of particles which could be seen under the microscope was betwee 10 000 and 20 000 per cu cm. Table 8 shows the weighted average size o particles in the water samples taken from the filter during Runs 6 and Eliassen estimated that the floc particles deposited in the filter contained 96 moisture and that about 8.5% of the dry solids was iron.

The average size of pores through which floc must pass is probably abou 5 to 10% of the sand size. The size of the pores in these experiments, if th estimate is correct, is three to four times the average size of the floc particle

Although most of the removal is doubtless accomplished by simple straining, the foregoing reflection leads to the conclusion that flocculation also takes place within the bed and is probably an important influence on the removal in the upper portion of the sand before the clogging becomes too great.

TABLE 8.—SIZE OF FLOC PARTICLES IN WATER FROM FILTER BED; WEIGHTED AVERAGE, IN MICRONS

Depth of sampling point, in feet	RUN 6	RUN 8	
	After 90 hr	After 47 hr	After 95 hr
Above filter	11	10.5	9.6
0.14	9.1	8.1	8.0
0.39	8.5	7.7	7.5
0.89	7.9	6.3	6.3
1.39	7.7	*	*
Effluent	6.3	*	*

* Only one or two small particles were visible in the field of 1 sq mm.

Conclusion.—The foregoing remarks should serve to indicate the complexity of the problem which the Committee is engaged in studying. Considerable research is in order for a future program. A good start has been made, and the work should be continued with the full support of the Sanitary Engineering Division.

WILLARD C. LAWRENCE,[17] ESQ. (by letter).[17a]—Lake Erie water has an average yearly turbidity of approximately 10 ppm, with a maximum of 100 ppm. This turbidity is a very finely divided clay silt, nearly collodial in character, and very difficult to precipitate. The raw water being so clear and the lake being so shallow, the micro-organism count at the Baldwin Filtration Plant, in Cleveland, Ohio, averages daily more than 1 000. For these reasons it is imperative to use not less than 0.8 grain of aluminum sulfate per gal and as much as 1.25 grains per gal due to the clay being difficult to coagulate. With the necessary proportion of chemicals applied to the raw water, with perfect mixing and coagulation, and with proper settling so as to have a large-sized floc applied to sand beds, a turbidity of 3 ppm or 4 ppm is proper.

Mentioning the character of Lake Erie water, and the treatment necessary, the writer believes that a hard round silica sand of approximately 0.6 mm size, and a uniformity coefficient of 1.2 to 1.6, should be used. The report recommends larger sizes of sand grains than are now commonly used and with this the writer agrees; but on Great Lakes waters they cannot be much greater than 0.6 mm, whereas on other waters they might be larger than 1.0 mm.

Experience at Cleveland indicates that one size of sand in a filter is all that is necessary, except a larger size for a supporting layer between the gravel and sand; or a very fine torpedo gravel layer at least 6 in. thick.

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[17] Supt. of Filtration, Baldwin Filtration Plant, Cleveland, Ohio.

[17a] Received by the Secretary February 25, 1937.

With the proper floc applied to a filter bed of 0.6 mm effective size, the sand bed need not be deeper than 12 in. With a shallower bed than the average, or the customary 24-in. bed, sand would probably not have to be removed and washed, treated with alkalies or acids, or require the installation of subsurface washing equipment, because, at the Baldwin Plant, the penetration of the floc is between 0.5 in. and 0.75 in,; so, with the properly washed filter beds, dirty sand should not accumulate. Since April 1, 1929, a variable wash-water rate at the Baldwin Plant, giving at least a 50% sand expansion at all seasons of the year, has been found to keep the filter sand clean.

At the time Messrs. Roberts Hulbert and Frank W. Herring gave their experimental studies of rapid filter washings at Detroit, Mich., in the fall of 1929, the writer discussed their paper[6] by presenting facts observed while using a 50% sand expansion that year on the entire Baldwin Plant, and the experimental results and practical results both showed that a 50% sand expansion is necessary for clean filter beds. In comparing sand analyses the writer has found that the sand at the Baldwin Plant which prior to 1929 had an effective size of 0.38 mm and a uniformity coefficient of 1.6, now (1937) has the same effective size and a uniformity coefficient of 1.25.

The Committee is correct in its statement that, in washing a filter, irrespective of the sand sizes, the filter wash should show only about one-half the rate of rise at the beginning of the wash to get the scouring effect before increasing the wash to its full velocity to carry off the dirt and organic matter. There is no question but that larger sand sizes yield longer runs, and reduce cost of operation without affecting the quality of the water produced if the floc is of large enough diameter so as not to penetrate the sand beds, but remains as a blanket on the top of the sand. The sand must be of smaller diameter than the floc produced. Sand sizes can be determined easily if the size of the floc produced by the particular water is known. The dirty sand beds are caused principally by: (*a*) Improperly constructed wash-water troughs (which cause a "hang back" of the dirt and organic matter); (*b*) insufficient, or unequal, wash-water pressures; (*c*) improperly designed under-drain systems; (*d*) a too shallow supporting layer of fine gravel, or large sand between the gravel bed and the sand bed so that dirt cannot penetrate into the gravel while the sand is being washed and in suspension; and (*e*) lack of care in grading the gravel bed.

The writer wishes to express to this Committee his interest in, and appreciation of, its studies.

G. F. GILKISON,[18] ESQ. (by letter).[18a]—The experimental filters in Kansas City, Mo., were constructed and operated in accordance with the instructions issued by the Committee. The work was very interesting, and some valuable information for the routine operation of the regular filter plant was obtained. When the results obtained at Kansas City are compared with those of the other co-operating cities, it seems almost unbelievable that the same equipment and technique were used by all operators. The writer spent considerable time trying either to correct, or to justify, this variation. He found that it was impossible to correlate his results with those of the other cities, and, frankly, was unable to justify them completely. Nevertheless, the following facts are noteworthy.

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[6] *Journal*, Am. Water Works Assoc., Vol. 21, November, 1929.

[18] Supt. of Filtration, Filtration Plant, Kansas City, Mo.

[18a] Received by the Secretary, March 5, 1937.

Turbidity of Applied Water.—The turbidity of the applied water averages 30 ppm with variations from 5 to 50 ppm. Changes occur very rapidly in the turbidity of the applied water.

Variation in Filter Head.—When the final coagulation basins are full there is a total head of 11 ft on the filters, 3 ft of which is negative head. Due to the fact that both the low-lift and secondary pumps are operated as constant speed pumps, there is a variation in the quantity of water pumped from the river and that delivered to the city. Frequent pump changes are avoided by varying the elevation of the water in the final basins. This change of elevation, which varies through a range of more than 2½ ft, is directly transferred to the elevation of the water applied to the filters. A change in the applied head naturally changes the loss-of-head reading.

These two variables would be expected to be reflected in the test filter operation and, undoubtedly, they were, but the fact remains that the regular filters of the Kansas City plant met these same variables, during the year the tests were being made, and the following average plant record was established:

Turbidity of applied water, in parts per million........	32
Turbidity of filtered water, in parts per million.......	0.13
Rate filtered, in million gallons daily per acre........	114.45
Filter run, in hours.............................	59.7
Percentage of filter water used for washing...........	1.15

It is indeed unfortunate that Kansas City was the only one in the co-operating group in which the applied water has a turbidity factor of more than 15 ppm and that it is practically free from micro-organisms. Had the group included several cities which have the Missouri River as a source of supply, or some other highly turbid supply, the results obtained by them would vary markedly from those obtained where a comparatively clear supply with high micro-organism content is used. The failure of the results at Kansas City to "fall in line" is not entirely due to personal error.

L. F. ALLAN.[19] ESQ. (by letter).[19a]—A masterly summary of experiments with small glass filters, in various cities, is contained in this report, which deals only with well established phenomena. Although the earlier experiments with graded sand media furnished valuable experience and produced considerable information of a general nature, no definite or absolute values for the various filtering and washing characteristics could be determined in a useful form. The experiments on uniform grades of sand were devised to provide this information, and, when the coefficients for all the characteristics of the various grades of uniform sand-filter media have been determined for all conditions of the raw water, practically all the information necessary for the design of a filter will be available. The report supplies much information and points the way for further research. Much work remains to be done to corroborate, or amend, and add, to the existing data and for this purpose it is suggested that the grades of uniform sand and the methods of preparing them for experiment be standardized.

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[19] Asst. Chf. Engr. of Water Supply, Toronto, Ont., Canada.

[19a] Received by the Secretary March 9, 1937.

The advantage of investigating the characteristics of uniform grades of filter sand is obvious, as it is in acordance with common engineering practice of establishing the properties of parts before considering the whole. It is probable that formulas will be found eventually to relate the properties of the uniform grades with those of any combination of grades. The formulas contained in the report (Equations (2) and (3)) afford a means of predicting the depth of filter medium required for any grading of sand that may be used. A similar formula concerning sand-rise characteristics was published by this writer[20] in 1935.

At the time when the experiments in Toronto were abandoned there were several avenues of investigation yet to be explored before the answer to the problem would be complete. However, the results of the work provide considerable material of value for filter-bed design. Eight filter runs seem to indicate a relationship between hours of service and size of sand grain which is not far from a straight line. On account of great variations in the quality of the raw water during the runs this relationship could not be determined exactly. Comparison of the chart of the plotted results with the penetration curve indicates that the length of a filter run is proportional to the depth of sediment penetration, irrespective of the size of particles. Here, again, some results are contradictory, but the general tendency is indicated.

Plotted results of sediment-penetration experiments give a smooth maximum penetration (critical depth) curve with the exception of one grade which is "out of line." This would seem to indicate the existence of a relationship between critical depth and size of sand grain. The determination of critical depths should include a complete cycle of water conditions and the values will vary in different localities.

The distribution of sediment in a filter after a run would be visualized more readily if the observed quantity of sediment in each segment of sand were plotted as a "concentration of turbidity" instead of "accumulated turbidity." When plotted as turbidi-milligrams per inch" against depth of the segment below the surface of the filter bed, the resultant curve, with its ordinates, encloses an area which represents the total quantity of turbidity, or sediment, in the filter from the surface to any depth.

The writer is of the opinion that, aside from economic considerations, uniform sand for filter media will give better results than graded sand,

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[20] *Journal,* Am. Water Works Assoc., February, 1935, p. 213.

and that, for the same top size, uniform sand beds need not be so deep. Sediment-distribution curves show that the greater part of the suspended matter is removed in the upper few inches of sand, so that the lower layers should be finer than the upper layers, if that were possible.

It is suggested that a combination consisting of a layer of moderately fine sand overlaid with a layer of coarse anthracite coal might be obtained such that, during back-washing, the sand would remain below the coal, and that this type of filter would be superior to either sand or coal.

Coarse grades of sand give long runs with satisfactory effluents, and wash better than finer grades. It may be thought that sand, coarser than that in general use, cannot be used in existing filters, but this is not always true because a uniform grade, considerably coarser than the top size of a graded bed, may be substituted without increasing the required depth. For instance, at both Toronto and Baltimore, it was found that 12 in. of a 0.75-mm particle is sufficient for the filter medium. If necessary the depth of the bed could be increased somewhat as the sand expansion would not be so great for coarse sand.

In conclusion, attention is directed to the fact that the experimental data have been secured under laboratory conditions, and suitable allowances must be made for correlation with actual conditions.

O. J. Ripple,[21] Esq. (by letter).[21a]—Under the heading, "Design Factors," the Committee recommends that, in the future, somewhat greater attention be given to the selection of the filtering material, and that other factors entering into filter-plant design be given more careful consideration; for example, it is stated that "initial or construction costs as well as operation and maintenance costs are important and frequently are the deciding factors in the selection of the filtering materials." More careful consideration should also be given both as to the grading of the material and as to the material itself, because, the writer believes, anthracite coal and possibly other filter materials will come into more general use in the next few years. The writer had hoped to have a combination of anthracite coal and sand accepted in the design of the West Side Filter Plant, in Denver, Colo., but the consulting engineers chose sand; nevertheless, his confidence in anthracite coal is not shaken.

The general trend toward larger sizes and more uniformly graded filter materials are justified, as stated in the report, because with better facilities for the preparation of the water for filtering and the added safety due to chlorination, advantage can be taken of longer runs obtained with the coarser sands. This report should tend to hasten this trend.

Another valuable feature of the report has to do with "critical depth" and this subject will bear further investigation and experimental work. Experiments along this line before the filter bed is designed should be economical. Under "Washing Filters: Two-Period Washing," a two-period water wash is given credit for increased efficiency. An air wash used before the water wash is more effective than any of the proposed surface, sub-surface, or "what have you" washes, that are mentioned so frequently in current literature. The writer has operated filter plants, both with and without air-washing facilities, and his experience has been in favor of the air wash. He has also used an air wash on some of the small glass filter experiments in a battery of filters set up under the specifications recommended by the Committee, and found that it was effective in removing mud-balls from sand as well as from anthracite coal.

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[21] Supt. of Filtration, Board of Water Commrs., Denver, Colo.

[21a] Received by the Secretary March 15, 1937.

In the absence of air-washing facilities or some form of the more expensive surface or sub-surface washes the two-period wash will give better efficiency, not so much in removing mud-balls but in cleaning the filter, because, as stated in the report, too high a rate of wash expands the sand so much that decreased impacts between sand grains lessens the scouring and cleaning action; however, the increased rate at the end of the wash does remove the loosened turbidity more effectively. The increase that is to be used will be limited by the free-board available and will vary with the wash-water temperature and size of the filter material.

The last series of experiments with sand of uniform size are very interesting and are in agreement with some experimental work conducted in the filter plants, at Denver, using a combination of coarse anthracite coal of uniform size supported on a bed of finer graded sand. The main reason for the supporting sand bed is to reduce the depth of the coal bed. Little progress has been made toward finding a better method of designating the size of the filtering materials, as the Hazen method, including the sieve analysis and graph, gives as plain a "picture" of the filtering materials as any of the proposed changes and with less necessary computation. Not more than four simple percentages or size designations are needed to obtain a satisfactory filter material, and a sieve analysis and graph are necessary to check it, no matter how the specifications read.

Much valuable knowledge as to the action of filtering materials, relative to penetration of floc, formation and prevention of mud-balls, rise of sand during washes and hydraulic grading, sizes to be used, etc., has been gained as a result of the activities of the members of this Committee and too much credit cannot be given them for their accomplishments.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

FLOW CHARACTERISTICS IN ELBOW DRAFT-TUBES[1]

Discussion

BY F. T. MAVIS, M. AM. SOC. C. E.

F. T. MAVIS,[4] M. AM. SOC. C. E. (by letter).[4a]—The author is to be commended for his painstaking work in constructing the model bends and in performing the tests which have been briefly summarized in this paper. He has demonstrated remarkable skill in making transparent model draft-tubes and in shaping and assembling the pyralin sections to be tested.

A quarter-turn draft-tube is no simple conduit. It may be visualized as a combination of a bend and an expanding section of pipe which changes not only in cross-sectional area but in shape as well. This combination, however, is not one that can be investigated readily by studying the peculiar behavior of each part and superposing the flow conditions which would obtain in each part separately. Each of these elements (the bend and the expanding section) has been studied by investigators over a period of more than 60 yr. There is still much to be learned about the behavior of water as it flows in a curved channel and in an expanding conduit, and the investigation of flow in draft-tubes has little more than made its "debut" in the hydraulic laboratory.

The writer proposes to confine his discussion to certain phases of the flow of water in curved pipes which have substantially the same cross-sectional area at the inlet and at the outlet. He will not attempt to discuss the hydraulics of expanding sections nor to dwell upon the combination of bend and diffuser in a draft-tube of the quarter-bend type.

Professor Mockmore has referred to the comprehensive tests on the flow of water in bends which were performed by Messrs. Yarnell and Nagler (22).[4b] With reference to investigations of head loss in bends, mention should be

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NOTE.—The paper by C. A. Mockmore, M. Am. Soc. C. E., was published in February, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] Prof. and Head, Mechanics and Hydraulics, and Cons. Engr., Iowa Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

[4a] Received by the Secretary April 28, 1936.

[4b] Numerals in parenthesis, thus (22), refer to corresponding numbers in Appendix I of the paper, *Proceedings*, Am. Soc. C. E., February, 1937, p. 283.

made of the pioneer work of Weisbach[5] in Germany, about the middle of the Nineteenth Century; and of the work of Alexander[6] and Brightmore[7] in England; and Williams, Hubbell, and Fenkell,[8] and Schoder[9] in the United States twenty-five to thirty years ago. More recently, investigations on the flow of water in curved pipes and miter bends have been carried on in Germany by Hofmann,[10] Schubart,[11] and others,[12] working in the Hydraulic Institute, at Munich; in Danzig, by Nippert,[13] and in the United States, by Giesecke,[14] by the staff of the National Hydraulic Laboratory, under the direction of Mr. H. N. Eaton, and by Wallace M. Lansford, Assoc. M. Am. Soc. C. E.,[15] at the University of Illinois. Most of these experiments were conducted on pipes of circular cross-section, although some were made on pipes of square or rectangular section. There is a mass of experimental data which could be collected from many published and unpublished investigations. It would be worth considerable to have these data brought together in a single report, and to have them re-appraised and correlated on the basis of present knowledge of the behavior of water as it flows around a bend.

The problem of the hydraulics of bends has also been investigated in some detail from an analytical, or mathematical, basis. Zur Nedden[16] in a paper on induced currents in fluids published in 1916, presented certain aspects of the problems which merit the mention of his paper in particular. He calls attention to the fact that a bend may perform quite differently depending on whether the flow of the fluid through it is laminar or turbulent. He cites specifically the case of a bend of rectangular cross-section, the inner and outer curves of which were hyperbolic, as causing a lower head loss than a circular bend as long as the flow remained laminar; and that as soon as the flow became turbulent, the hyperbolic bend caused a greater head loss than the circular bend. This behavior, of course, would be apparent if the coefficients of velocity head, representing the excess head loss in the bend, were plotted as a function of Reynolds' number. The particular bend to which Zur Nedden refers was one tested by Grether[17] which in plan was similar to the bend which Professor Mockmore calls Bend No. 2.

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[5] "Lehrbuch der Ingenieur und Maschinen Mechanik", von J. Weisbach, Leipzig, 1845, p. 439.

[6] "The Resistance Offered to the Flow of Water in Pipes by Bends and Elbows", by C. W. L. Alexander, *Minutes of Proceedings*, Inst. C. E., Vol. 159, pp. 341–364, 1904–05, Pt. 1.

[7] "Loss of Pressure in Water Flowing Through Straight and Curved Pipes", by A. W. Brightmore, *Minutes of Proceedings*, Inst. C. E., Vol. 169, p. 315, 1907.

[8] "Experiments at Detroit, Mich., on the Effect of Curvature on the Flow of Water in Pipes", by Gardner S. Williams, C. W. Hubbell, and G. H. Fenkell, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. XLVII (1902), pp. 1–369.

[9] "Curve Resistance in Water Pipes", by E. W. Schoder, *Transactions*, Am. Soc. C. E., Vol. LXII (1907), p. 67.

[10] "Neue Untersuchungen über den Druckverlust in Rohrkrümmern", von A. Hofmann, *Mitteilungen* des Hydraulischen Instituts der Technische Hochschule, Munchen, Germany, Heft 2, pp. 70–71, 1928; also, Heft 3, p. 45 *et seq.*, 1929.

[11] "Energy Loss in Smooth- and Rough-Surfaced Bends and Curves in Pipe Lines", by W. Schubart, (tr. by F. T. Mavis), *Transactions*, Hydr. Inst. of the Munich Technical Univ., *Bulletin 3*, pp. 81–99 (A. S. M. E. 1935).

[12] "Verluste in glatten Rohrkrümmern mit kreisrundem Querschnitt bei weniger als 90° Ablenkung", von R. Wasielewski, *Mitteilungen* des Hydr. Inst. der Techn. Hochschule, München, Heft 5, pp. 53–67, 1932.

[13] Uber den Strömungsverlust in gerkrümmten Kanälen,", von H. Nippert, *Forschungsarbeiten*, Heft 32 (1929).

[14] "Friction of Water in Elbows", by F. E. Giesecke, M. Am. Soc. C. E., *Transactions*, Am. Soc. for Heating and Ventilating Engrs., Vol. 32, pp. 303–314, 1926.

[15] "The Use of an Elbow in a Pipe for Determining the Rate of Flow in the Pipe", by Wallace M. Lansford, *Bulletin No. 289*, Eng. Experiment Station, Univ. of Illinois, 1936.

[16] "Induced Currents in Fluids", by F. Zur Nedden, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), pp. 844–913.

To-day, it is generally recognized that a liquid, in flowing around a bend under turbulent flow conditions, moves in a so-called "spiral" or "helical" path, and that the induced currents and attendant impact are largely responsible for excess losses of head in bends. In 1902, the late H. W. Brinckerhoff, M. Am. Soc. C. E., discussing[18] a paper by Clarence W. Hubbell, George H. Fenkell, and the late Gardner S. Williams, Members, Am. Soc. C. E., explained the fact that a bend offers greater resistance than an equal length of straight pipe on a hypothesis of spiral flow which he described in its essentials precisely as Professor Mockmore and other observers have done. In their closing discussion, the authors of this paper replied to Mr. Brinckerhoff's comments as follows:

> " * * * at one time, one of them [the authors] was disposed to explain certain phenomena on the theory of a spiral motion, but in spite of attempts to locate such motion, no direct experimental evidence of its existence has yet been obtained, and, though the writers consider that Mr. Brinckerhoff's theory is ingenious and possibly correct, they are not prepared to endorse it."

Transparent model bends, such as those used by Professor Mockmore, permit the observer to see this spiral or helical flow. These transparent models have done much to promote a better understanding and a more intimate knowledge of the phenomena of hydraulics. To-day, this spiral flow in bends can be observed; three decades ago, it was an hypothesis which eminent hydraulic engineers were then " * * * not prepared to endorse * * *."

The intensity of induced currents depends chiefly upon the pressure gradient along a section which is normal to the direction of motion of particles of water in the bends. With slight modification, Equation (4) can be written as follows:

$$p_2 - p_1 = \frac{w}{g} \int_{r_1}^{r_2} V^2 \frac{dr}{r} \dots\dots\dots\dots\dots\dots (26)$$

in which $p_2 - p_1$ = the difference in pressure at Points (2) and (1) on a section normal to the flow of water particles; r_2 and r_1 = the radii of curvature of stream filaments at Points (2) and (1), respectively; V = the velocity of flow along any filament the radius of curvature of which is r (and V is, therefore, a function of r); and, $\frac{w}{g}$ = the specific mass of water.

If the width of bend $(r_2 - r_1)$ is small in proportion to the radius of curvature, 0.5 $(r_2 + r_1)$, and if the depth of the channel is large in a direction normal to the plane of curvature; that is, several times $r_2 - r_1$, the velocity will probably be nearly constant and it may be replaced by the average velocity, V_{av}. Under these conditions it may be justifiable to state as a fair approximation that the difference in pressure on the rigid side-walls which are a distance ($r_2 - r_1$) apart will be, as given by Equation (4),

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[17] "Krümmer die Potentialbewegung zulassen", von H. Grether, *Verhandlung* d. Ver. z. Beförderung d. Gewerbefl., 1909, p. 117 ff.

[18] *Transactions*, Am. Soc. C. E., Vol. XLVII (1902), p. 272.

$$p_2 - p_1 = \frac{w}{g} V^2_{av} \log_e \left(\frac{r_2}{r_1} \right) \dots\dots\dots\dots\dots\dots (27)$$

Co-axial guide-vanes, in general similar to the horizontal partition in Draft-Tube No. 2, may be expected to reduce the head loss in bends for three reasons: (1) The vanes tend to act as baffles and equalize the velocity distribution; (2) by reducing the ratio, $\frac{r_2}{r_1}$, for the individual channels and making it nearer unity than the corresponding ratio for the bend as a whole, the pressure difference, $p_2 - p_1$, is reduced between any two given vanes; and (3), the unbalanced force exerted on a given guide-vane is distributed over a curved plate supported on the side walls of the bend. The efficacy of guide-vanes in reducing losses due to the flow of air in bends has been demonstrated, and bends fitted with co-axial guides have been used in wind tunnels of the return-flow type. In principle, they should also be effective in water conduits although there are certain apparent obstacles in the way of obtaining structural strength at costs low enough to justify the construction of guide-vanes.

The phenomena of flow of water through bends, expanding conduits, and draft-tubes, deserve intensive study in model and prototype. Professor Mockmore's approach through the medium of transparent models leads the hydraulic engineer a long way toward a better understanding of those phenomena.

The following corrections will be made in the paper for *Transactions*: Delete the first sentence following Equation (7); page 258, Fig. 3(*b*), delete the equation, "$y = \frac{x - 2^3}{3d^2}$"; and page 265, lines 23 and 24, delete the phrase, "of the velocity head."

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

A NEW THEORY OF RAIL EXPANSION

Discussion

BY MESSRS. CHESTER F. GAILOR, AND E. F. KENNEY

CHESTER F. GAILOR,[10] ESQ. (by letter).[10a]—The calculations and the formula presented in this paper appear to be correct and should offer an excellent basis for continuing the study of this subject, as it is very evident that the information available heretofore is inadequate for modern track construction[10b]. The following factors entering into this subject should be taken into consideration in future studies and should be given adequate thought by engineers preparing to install rail in long continuous lengths:

(1) Most, if not all, of the rail laid in long lengths in the United States has been installed during the summer months when the expansion of the rail was greatest; therefore, the objections or complications from a practical point of view which might arise from this type of construction are vastly different from those that would arise if the same length of track was laid when the rails were shortest.

(2) Mr. Africano assumes that these long lengths of rail are laid in a perfectly straight line and that this alignment is so maintained. This is not exactly the case as can be ascertained easily and quickly by observing, with an instrument during a very hot day, the lateral variation in the alignment, to say nothing of the possible vertical movement that may occur in a similar manner.

(3) The free ends of long lengths of continuous rail will not expand any appreciable amount provided the rail is adequately fastened at each tie. This fact seems to have been clearly proved on the Delaware and Hudson installations.

(4) It would be well to emphasize the necessity of holding the rail definitely at each tie by means of a fastener that will always maintain

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NOTE.—The paper by Alfred Africano, Jun. Am. Soc. C. E., was published in February, 1937, *Porceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[10] Cons. Engr., New York, N. Y.

[10a] Received by the Secretary, March 13, 1937.

[10b] Corrections for *Transactions*, approved by author: In caption to Fig. 2(*d*), change "Ends" to read "Ties"; in Fig. 3, change ordinate, "$s + \frac{Tx}{c}$" to "$P + \frac{Tx}{c}$"; in Fig. 3, change ordinate "$P = E\ n\ \Delta\ t$" to "$s = E\ n\ \Delta\ t$"; and in the two lines above Equation (4) delete "(Equation (37))".

a pre-determined gripping strength such as the spring fastenings exert on the rails laid on the Delaware and Hudson Railroad. The gripping strength required on this type of construction, is maintained at 1 500 to 2 500-lb pressure on the base of the rail by each spring fastener. It would appear that disastrous results might be expected if long lengths of rails were installed without similarly fastening the rails at each tie.

(5) The welding of long lengths of rails on electric railway systems has been in practice for many years and the question of expansion or contraction when rails are laid in paved streets is a subject which, to-day, requires only routine attention. However, the fact that the head of rails embedded in street pavement does expand and move laterally can be seen by observing, on a hot day, the effects of this action along the outside of the head of the rail, especially in asphalt pavements. This action is noticeable, even if the base of the rail is embedded firmly in concrete and, necessarily, not subject to such variations in temperature as are found in the head of the rail.[11]

(6) The expansion or contraction of rails might well be better analyzed by assuming that fastenings at each tie are sufficient to hold all pressures or stresses that may develop between such fastenings. In practice, there is little or no evidence that these pressures or stresses are accumulative.

(7) There is need of more definite information on this subject, which should be obtained from a series of practical and laboratory tests, as the practice of laying long lengths of rail on railroad systems is rapidly becoming standard.

Mr. Africano should be commended for the thoroughness of his study and for the value of the information contained in his paper.

E. F. KENNEY,[12] ESQ. (by letter).[12a]—The vital feature in this subject of rail expansion is not so much the control of the linear expansion as the control of the lateral movement which might result unless very substantial fastenings are applied to the rail. It has been shown that some types of fastening are ample to take care of this feature, but it is not so certain that the fastenings used on the average railroad will do so safely. Thinking a little further, one wonders what would be the outcome when economic conditions affect the upkeep, even if fastenings are used which, when well maintained, are capable of holding the alignment.

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[11] See *Proceedings*, Am. Transit Eng. Assoc. (formerly Elec. Ry. Eng. Assoc.), for information on this subject.

[12] Metallurgical Engr., Bethlehem Steel Co. (Inc), Bethlehem, Pa.

[12a] Received by the Secretary March 1, 1937.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ECONOMICS OF HIGHWAY-BRIDGE FLOORINGS OF VARIOUS UNIT WEIGHTS

Discussion

BY JONATHAN JONES, M. AM. SOC. C. E.

JONATHAN JONES,[3] M. AM. SOC. C. E. (by letter).[3a]—It should only emphasize the high regard in which the writer holds this paper, to state that its value will ultimately lie more in the method of approach and analysis which it sets forth, than in its statistical data. To the writer, it seems impossible to generalize the problem so that the unit costs involved will lie in the same pattern to-morrow as to-day; even the basic assumption that unit costs of flooring will be somewhat in inverse ratio to their unit weights, may not always hold.

Furthermore, all the factors that have been carefully analyzed in the paper for the general, may not enter into the particular, case. The most interesting case in which, within the writer's knowledge, the designing engineer erred in selecting a light-weight and expensive floor, involved several narrow 210-ft truss spans on adequate old piers. The cost of the piers, therefore, was not a factor in the problem, and to that extent at least the application of Fig. 5 would have been in error. Had the designing engineer, however, applied the logic of the author's method to those factors which did exist in his problem, and to the unit costs which he could foresee at his location, he would inevitably have found that he would have saved several thousand dollars by adopting a "standard" or even a "super-standard" floor.

The relationship between 4 cts per lb for secondary joists and 7 cts or more for an added stringer, would similarly be debatable in a given case. From such considerations comes the writer's feeling that, although the method herein given is invaluable, the most exact data should be applied to each particular case, and the author's data should be considered as illustrative.

NOTE.—The paper by J. A. L Waddell, Hon. M. Am. Soc. C. E., was published in February, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[3] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.

[3a] Received by the Secretary March 16, 1937.

The important effect of a variation in the data may be demonstrated by reference to Table 3. Using the author's unit costs,[3b] the minimum simple span at which the "standard" and the "50-lb" floor will be of equal cost

TABLE 3.—Effect of a Variation in Data, in the Case of Simple-Truss Bridges

Item No.	Description	20-Foot Roadway		40-Foot Roadway	
		Carbon steel	Silicon steel	Carbon steel	Silicon steel
	(*a*) Spans of Equal Cost, Using the Author's Data				
1	Standard 9-in., 110-lb slab	$19.60	$19.60	$39.60	$39.60
2	Special 50-lb floor	34.20	34.80	71.20	72.55
3	Difference	$14.60	$15.20	$31.60	$32.95
4	From Fig.	5	6	1	2
5	Read the span length of equal cost, in feet	300	380	270	335
	(*b*) Spans of Equal Cost, if Cost of the 50-Pound Floor Is Reduced from $1.30 to $1.15 per Square Foot				
6	Difference is less than Item No. 3 by:	$3.00	$3.00	$6.75	$6.75
7	New difference (Item No. 3 less than Item No. 6)	$11.60	$12.20	$24.85	$26.20
8	Span length of equal cost, in feet	230	320	180	240

is shown in Table 3(*a*); and in Table 3(*b*) these spans are re-calculated on the hypothesis that the cost of the "50-lb" floor has been reduced from $1.30 to only $1.15 per sq ft.

There are two generalizations in the paper toward which the writer would suggest considerable caution; these are found under the heading, "Suspension Bridges." With respect to suspension bridges it is stated "the lighter the flooring the less will be the first cost of the structure." The first requirement in approaching the design of any suspension bridge is to place a maximum limit on its flexibility, or deformation under partial live load. The weight of the flooring, and the consequent weight of the cables, have an important influence on stiffness. Granted that the design has been established as just adequately stiff, using a "standard" floor, then, if it is decided to use a lighter flooring and less wire in the cables, it will follow that the structure exceeds the pre-determined limit of flexibility, and the stiffening trusses will have to be increased in section or depth, or both, to restore the rigidity that has been taken out. Some, at least, of the money saved on cables and other main structure, therefore, must be spent on the stiffening system; and this is rather a pity because a dollar spent in the main structure is better invested than a dollar spent for local stiffening. This type of structure, therefore, calls for a complete comparative analysis.

[3b] Corrections for *Transactions*, approved by author: In the fourth line below Fig. 8, change the sentence to read: "The cost per linear foot of structure for the two reinforced concrete guards and their supporting base is $2 for the 2 cu ft of concrete * * *"; on p. 316, the line preceding "Type *C* Cantilevers", change "535 lb" to read "53.5 lb"; in Table 2, Sub-Caption (*b*), change "Figs. 1 to 4" to read "Figs. 5 to 8"; and on p. 317, last line, change "Figs. 1 to 4" to read "Figs. 5 to 8".

With respect to cantilevers, the writer does not see the objection, from the æsthetic viewpoint, in using the heavier flooring on part or on all of the anchor arms. The change from the slab floor of the approaches to the special floor of the bridge spans must be made somewhere, and there seems to be no reason not to make it where it will be most beneficial. The prevention or, at all events, the minimizing, of live load reversal on the main truss-to-anchorage pins is proving to be extremely desirable on old cantilevers, even if it calls for longer anchor arms than former practice favored; to augment the downward force at this point by the use of heavy flooring on the anchor arms seems much more important than the question of where the tourist leaves the concrete and enters upon the metallic deck.

This paper is a most important contribution to one of the livest topics, from a money standpoint, now before the structural engineer, and thanks are due to Mr. Waddell for the effort expended upon it.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

RAINFALL INTENSITIES AND FREQUENCIES

Discussion

By Messrs. Victor H. Cochrane, and LeRoy K. Sherman

Victor H. Cochrane,[7] M. Am. Soc. C. E. (by letter).[7a]—The remarkably consistent results obtained by the authors and other investigators indicate that the hyperbolic, or Talbot, type of formula is correct for precipitation lasting two hours or less. For some years the writer has made use of a formula giving the intensities of precipitation for various periods in terms of I, the precipitation in 1 hr. If in a given case, I is known or assumed, the proper intensities for other periods from 5 to 120 min may be calculated from the formula,

$$i = \frac{60 + n\,I}{\frac{t}{I} + n} = \frac{(60 + n\,I)\,I}{t + n\,I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which n is a constant depending upon the frequency, F. Its value varies from approximately 8 for $F = 100$ to about 12 for $F = 1$. It may be assumed according to the formula,

$$n = 12 - 2 \log F \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Combining Equations (25) and 26):

$$i = \frac{60 + (12 - 2 \log F)\,I}{\frac{t}{I} + (12 - 2 \log F)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

If $F = 10$, $n = 10$, an average value. In case it is desired to use a value of I for which the frequency is unknown, this average value of n may be used for practical purposes.

Note.—The paper by A. J. Schafmayer, M. Am. Soc. C. E., and the late B. E. Grant, Esq., was published in February, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[7] Cons. Engr., Tulsa, Okla.

[7a] Received by the Secretary March 22, 1937.

Table 8 shows comparative results computed from Equations (18) and (27). The fact that hyperbolic curves fit rainfall data covering short periods suggests that a formula giving flood flows from small water-sheds should likewise be hyperbolic.

TABLE 8.—COMPARATIVE VALUES OF i.

Elapsed time, t, in minutes	FREQUENCY EQUALS ONCE IN EVERY:					
	2 yr.		10 yr.		100 yr.	
(1)	Equation (18) (2)	Equation (27) (3)	Equation (18) (4)	Equation (27) (5)	Equation (18) (6)	Equation (27) (7)
=	1.34	1.34	2.03	2.03	3.03	3.03
=	.....	11.4	.....	10.0	.....	8.0
120.....	0.75	0.75	1.18	1.16	1.79	1.77
100.....	0.88	0.88	1.37	1.36	2.07	2.05
60.....	1.34	1.34	2.03	2.03	3.03	3.03

Elapsed time, t, in minutes	FREQUENCY EQUALS ONCE IN EVERY:					
	2 yr.		10 yr.		100 yr.	
(1)	Equation (18) (2)	Equation (27) (3)	Equation (18) (4)	Equation (27) (5)	Equation (18) (6)	Equation (27) (7)
30.....	2.24	2.23	3.26	3.24	4.72	4.70
15.....	3.29	3.33	4.60	4.62	6.47	6.50
10.....	3.93	3.99	5.35	5.38	7.38	7.45
5.....	4.86	4.98	6.43	6.44	8.68	8.73
.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....

LEROY K. SHERMAN,[8] M. AM. SOC. C. E. (by letter).[8a]—Rainfall data are essential for the design of storm sewers and many other works. Analysis and interpretation of the data are of equal importance. The utilization of station-years in statistical methods, where such stations are near isohyetals for the same rainfull durations and frequencies, appears to be sound. Stations with materially different precipitation characteristics, such as Chicago, Ill., and Nashville, Tenn., should not be grouped together. Stations, such as St. Paul, Minn., Chicago, and Central New York State, all near the same isohyetal, may properly be placed in the same group.

The authors give preference to the hyperbola type of curve to express the rainfall relations. Others who have made similar studies prefer the exponential type of rainfall equation. In so far as the mathematical expression closely fits the data, the form of expression is relatively unimportant. The form which fits the widest range of observations is frequently the most desirable.

An examination of a large number of automatic rain-gage records in the District of Columbia[9] showed few instances of even approximate uniform rainfall rates for periods of 30 min or more. A fair average rainfall pattern was triangular in form, with time duration as the base and the peak rate at twice the average rate. These rain irregularities are partly "ironed out" on large areas during the time of run-off. Does the same apply to small urban areas? The authors' diagrams show that 2 in. of rain may fall during 1 hr: 1.3 in. in 20 min, and 0.7 in 40 min.

The rational method is now commonly used to estimate run-off from severed areas. The average hourly rate of rain for the period of concentration is used. It is pertinent to find what, if any, difference in run-off exists under the assumption of the average and the actual rates of rain.

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[8] Cons. Engr., Chicago, Ill.

[8a] Received by the Secretary March 22, 1937.

[9] "Elimination of Pollution of Rock Creek, August 1, 1935", by L. K. Sherman and W. W. Horner, Members, Am. Soc. C. E., Gov't. Printing Office, Washington, D. C.

The writer has determined this for a particular case—a severed are of 420 acres in Evanston, Ill., north of Chicago. The run-off characteristics of this area are given in a paper by S. A. Greeley,[10] M. Am. Soc. C. E The writer has derived the typical percentage distribution of run-off from this area so as to conform to Mr. Greeley's time-run-off data. He ha applied his own unit hydrograph method[11] and the same method a improved by Merrill M. Bernard,[12] M. Am. Soc. C. E., and by W. W Horner, M. Am. Soc. C. E., and F. L. Flynt,[13] Assoc. M. Am. Soc. C. E.

A 10-yr storm for 50 min was taken. By Fig. 5, the average intensit of rain per hour for 50 min is 2.34 in., or a total of 1.95 in.[13a] The initia

TABLE 9.—COMPUTATION OF RUN-OFF

Elapsed time, in 10-min. intervals	RAINFALL, IN INCHES OF DEPTH			Percentage, distribution of rainfall	DISTRIBUTION OF RUN-OFF, IN INCHES						RUN-OFF	
	Gage reading	Loss	Net rainfall								In inches	In cubic feet per
(1)	(2)	(3)	(4)	(5)	(6a)	(6b)	(6c)	(6d)	(6e)	(6f)	(7)	(8)
					(a) WITH UNIFORM RATES OF RAINFALL							
1.......	0.39	0.09	0.3	8	0.024	.....	.....	.....	.....	....	0.024	61
2.......	0.39	0.09	0.3	19	0.057	0.024	.....	.....	.....	....	0.081	206
3.......	0.39	0.09	0.3	28	0.084	0.057	0.024	.....	.....	....	0.165	419
4.......	0.39	0.09	0.3	23	0.069	0.084	0.057	0.024	.....	....	0.234	595
5.......	0.39	0.09	0.3	15	0.045	0.069	0.084	0.057	0.024	....	0.279	708
....	....	....	....	7	0.021	0.045	0.069	0.084	0.057	0.024	0.300	762
....	....	....	....	....	.....	0.021	0.045	0.069	0.084	0.057	0.276	701
....	....	....	....	....	.....	.....	0.021	0.045	0.069	0.084	0.219	556
....	....	....	....	....	.....	.....	.....	0.021	0.045	0.069	0.135	343
....	....	....	....	....	.....	.....	.....	.....	0.021	0.045	0.066	168
....	....	....	....	....	.....	.....	.....	.....	.....	0.021	0.021	53
....	1.95	0.45	1.50	100	0.30	0.30	0.30	0.30	0.30	0.30	1.8000	....
					(b) WITH NON-UNIFORM RATES OF RAINFALL							
1.......	0.22	0.09	0.13	8	0.0104	.....	.....	.....	.....	....	0.0104	25
2.......	0.22	0.09	0.13	19	0.0247	0.0104	.....	.....	.....	....	0.0351	92
3.......	0.40	0.09	0.31	28	0.0364	0.0247	0.0248	.....	.....	....	0.0359	218
4.......	0.90	0.09	0.81	23	0.0299	0.0364	0.0589	0.0648	.....	....	0.1900	482
5.......	0.21	0.09	0.12	15	0.0195	0.0299	0.0868	0.1539	0.0096	....	0.2997	760
....	....	....	....	7	0.0091	0.0195	0.0713	0.2268	0.0228	....	0.3495	890
....	....	....	....	....	.....	0.0091	0.0465	0.1863	0.0336	....	0.2755	700
....	....	....	....	....	.....	.....	0.0217	0.1215	0.0276	....	0.1708	434
....	....	....	....	....	.....	.....	.....	0.0567	0.0180	....	0.0747	190
....	....	....	....	....	.....	.....	.....	.....	0.0084	....	0.0084	21
....	1.95	0.45	1.50	100	0.13	0.13	0.31	0.81	0.12	....	1.5000	....

and infiltration loss was taken at 0.45 in. Under these conditions, the pea rate of run-off, by the rational method, is 755 cu ft per sec. Th peak run-off for this same case, computed by the unit hydrograph method is practically the same (762 cu ft per sec) as shown in Table 9(*a*). If the rain is continued beyond the time of concentration, the run-off rate (762 cu ft per sec, in Column (8), Table 9), will remain constant.

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[10] *Journal*, Western Soc. of Engrs., September, 1913; also, "Design of Sewers", b Metcalf and Eddy.

[11] "Runoff from Rainfall by the Unit Graph", *Engineering News-Record*, April 7, 1932

[12] "An Approach to Determinate Stream Flow", *Transactions*, Am. Soc. C. E., Vol. 10 (1935), p. 347.

[13] "Relation Between Rainfall and Run-Off from Small Urban Areas", *Transaction* Am. Soc. C. E., Vol. 101 (1936), p. 140.

[13a] Correction for *Transactions:* Caption of Fig. 4 change "65 Station-Years" to read "659 Station-Years", *Proceedings*, Am. Soc. C. E., February, 1937, p. 236.

The peak run-off for this non-uniform rate of rainfall is 890 cu ft per sec (Table 9(*b*)). This is about 17% greater than the run-off due to uniform rainfall. One may conclude that, for a given coefficient or a rate of loss, the rational method will forecast the minimum rate of run-off corresponding to the storm duration and frequency. On large natural areas, with flood-producing storms lasting for several days, and where two-thirds of the total rainfall commonly occurs in one day, the use of average storm rates is inadmissible.

All of the foregoing goes to show the importance of rainfall data segregated into divisions of duration-intensity-frequency, such as the authors have presented.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ANALYSIS OF STRESSES IN SUBAQUEOUS TUNNEL TUBES

Discussion

By David J. Peery, Jun. Am. Soc. C. E.

David J. Peery,[6] Jun. Am. Soc. C. E. (by letter).[6a]—The method described in this paper is quite convenient for purposes of computation and design, being a very simple and ingenious method for analyzing a structure which is statically indeterminate in the fourth degree. Equation (4), however, represents the horizontal deflection of the elastic center, rather than the deflection of a point on the ring. In order to avoid ambiguity in nomenclature, assume that the specific point at which the deflection is desired has the co-ordinates, x', y'. Other nomenclature will be as shown in Fig. 3, the variables, x and y, representing the co-ordinates to the curve of the neutral axis of the ring. The horizontal distortion at Point x', y', will be.

$$\Delta_{ou} = \int M\,(y - y')\,dw + \int H \cos^2\theta\,dv$$
$$+ \int V \sin\theta \cos\theta\,dv - H_o\Big[\int y\,(y$$
$$- y')\,dw + \int \cos^2\theta\,dv\Big] - M_o \int (y$$
$$- y')\,dw \;\ldots\ldots\ldots\ldots (28)$$

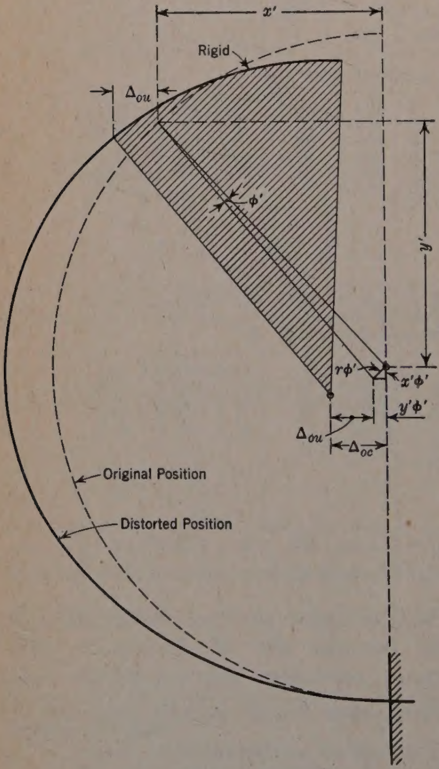

Fig. 6

Note.—The paper by A. A. Eremin, Assoc. M. Am. Soc. C. E., was published in December, 1936, *Proceedings*. This discussion is printed in Proceedings in order that the views expressed may be brought before all members for further discussion of the paper.

[6] Instructor, Missouri School of Mines, Rolla, Mo.

[6a] Received by the Secretary February 15, 1937.

the limits of integration being taken from the fixed support to Point x', y'. If the limits of integration are taken from the fixed support to the Point x', y', Equation (4) represents the horizontal deflection of the elastic center when the part of the ring between the fixed support and the Point x', y', is considered flexible and the other part of the ring is considered rigid. If this deflection is Δ_{oc}, Equation (4) and Equation (28) yield:

$$\Delta_{ou} = \Delta_{oc} - y' \int (M - H_o\, y - M_o)\, dw \ldots\ldots\ldots\ldots\ldots (29)$$

The term under the integral sign represents the angular rotation of the tangent at Point x', y'. Equation (4) will represent the horizontal deflection of Point x', y', only when there is no rotation of the tangent at the point, x', y'. This will not be true in a general case. If this rotation at Point x', y', is called ϕ', Equation (29) becomes:

$$\Delta_{ou} = \Delta_{oc} - y'\, \phi' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

The geometric significance of the last term in Equations (29) and (30) is evident from Fig. 6. Equations (8) and (27) of the paper will also have to be adjusted in the same manner.